

$$\begin{aligned}
\llbracket p_1 \cup p_2 \rrbracket_{fo}(x, y) &:= \llbracket p_1 \rrbracket_{fo}(x, y) \vee \llbracket p_2 \rrbracket_{fo}(x, y) \\
\llbracket /p \rrbracket_{fo}(x, y) &:= \exists z. \mathbf{root}(z) \wedge \llbracket p \rrbracket_{fo}(z, y) \\
\llbracket p_1/p_2 \rrbracket_{fo}(x, y) &:= \exists z. \llbracket p_1 \rrbracket_{fo}(x, z) \wedge \llbracket p_2 \rrbracket_{fo}(z, y) \\
\llbracket axis::test[q] \rrbracket_{fo}(x, y) &:= axis(x, y) \wedge \mathbf{lab}_{test}(y) \wedge \llbracket q \rrbracket_{fo}(y) \\
\llbracket q_1 \wedge q_2 \rrbracket_{fo}(x) &:= \llbracket q_1 \rrbracket_{fo}(x) \wedge \llbracket q_2 \rrbracket_{fo}(x) \\
\llbracket q_1 \vee q_2 \rrbracket_{fo}(x) &:= \llbracket q_1 \rrbracket_{fo}(x) \vee \llbracket q_2 \rrbracket_{fo}(x) \\
\llbracket \neg q \rrbracket_{fo}(x) &:= \neg \llbracket q \rrbracket_{fo}(x) \\
\llbracket \mathbf{true} \rrbracket_{fo}(x) &:= \mathbf{true} \\
\llbracket p \rrbracket_{fo}(x) &:= \exists y. \llbracket p \rrbracket_{fo}(x, y)
\end{aligned}$$

where:

$$\begin{aligned}
\mathbf{root}(x) &= \forall y. \neg \mathbf{child}(y, x) \\
\mathbf{lab}_*(y) &= \mathbf{true} \\
\mathbf{self}(x, y) &= (x = y) \\
\mathbf{descendant-or-self}(x, y) &= (x = y \vee \mathbf{descendant}(x, y)) \\
\mathbf{following}(x, y) &= \exists z, z'. (z = x \vee \mathbf{descendant}(z, x)) \wedge \mathbf{following-sibling}(z, z') \\
&\quad \wedge (z' = y \vee \mathbf{descendant}(z', y)) \\
axis^{-1}(x, y) &= axis(y, x)
\end{aligned}$$

Figure 1: Translation of CoreXPath into First-Order Logic.