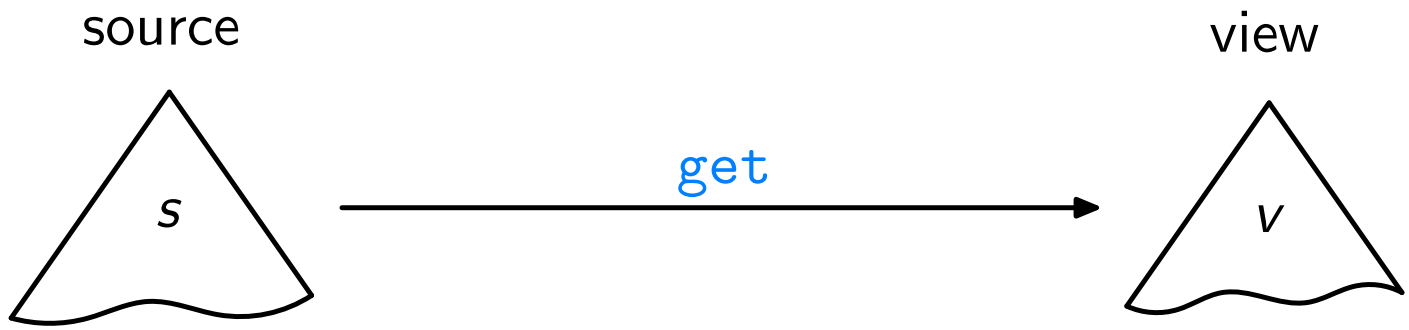
Bidirectionalization for Free!

Janis Voigtländer

Technische Universität Dresden

POPL'09

Bidirectional Transformation



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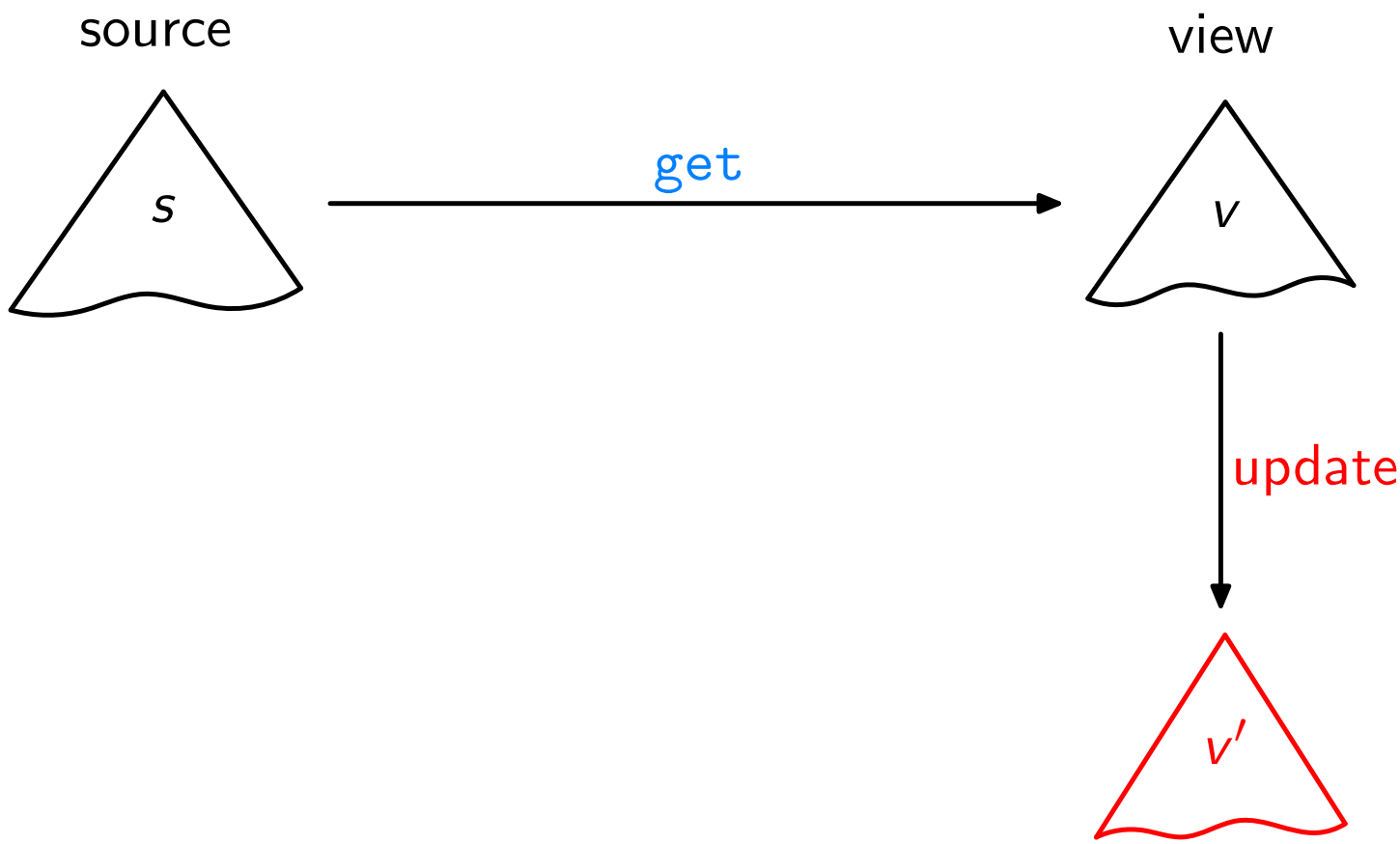

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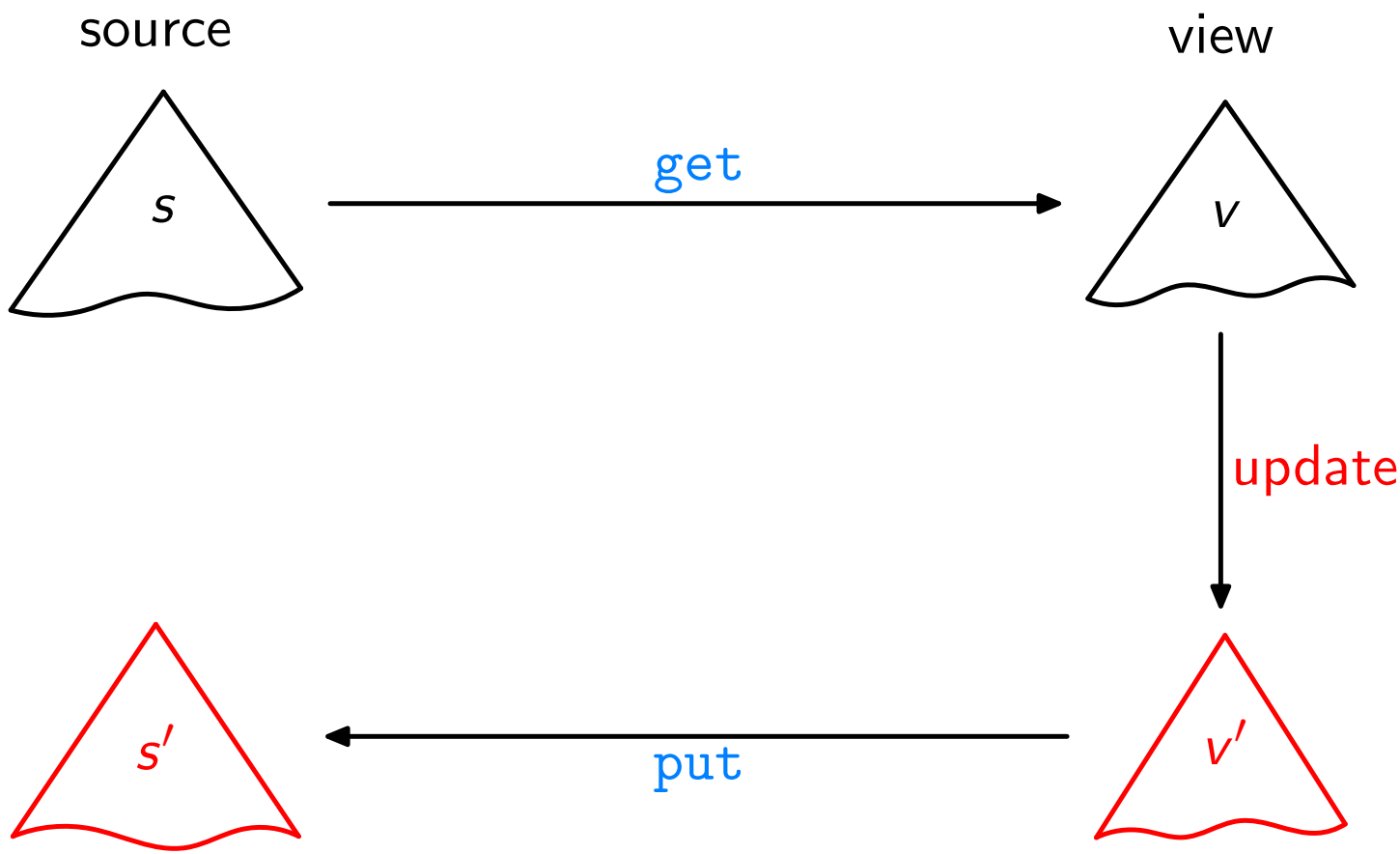

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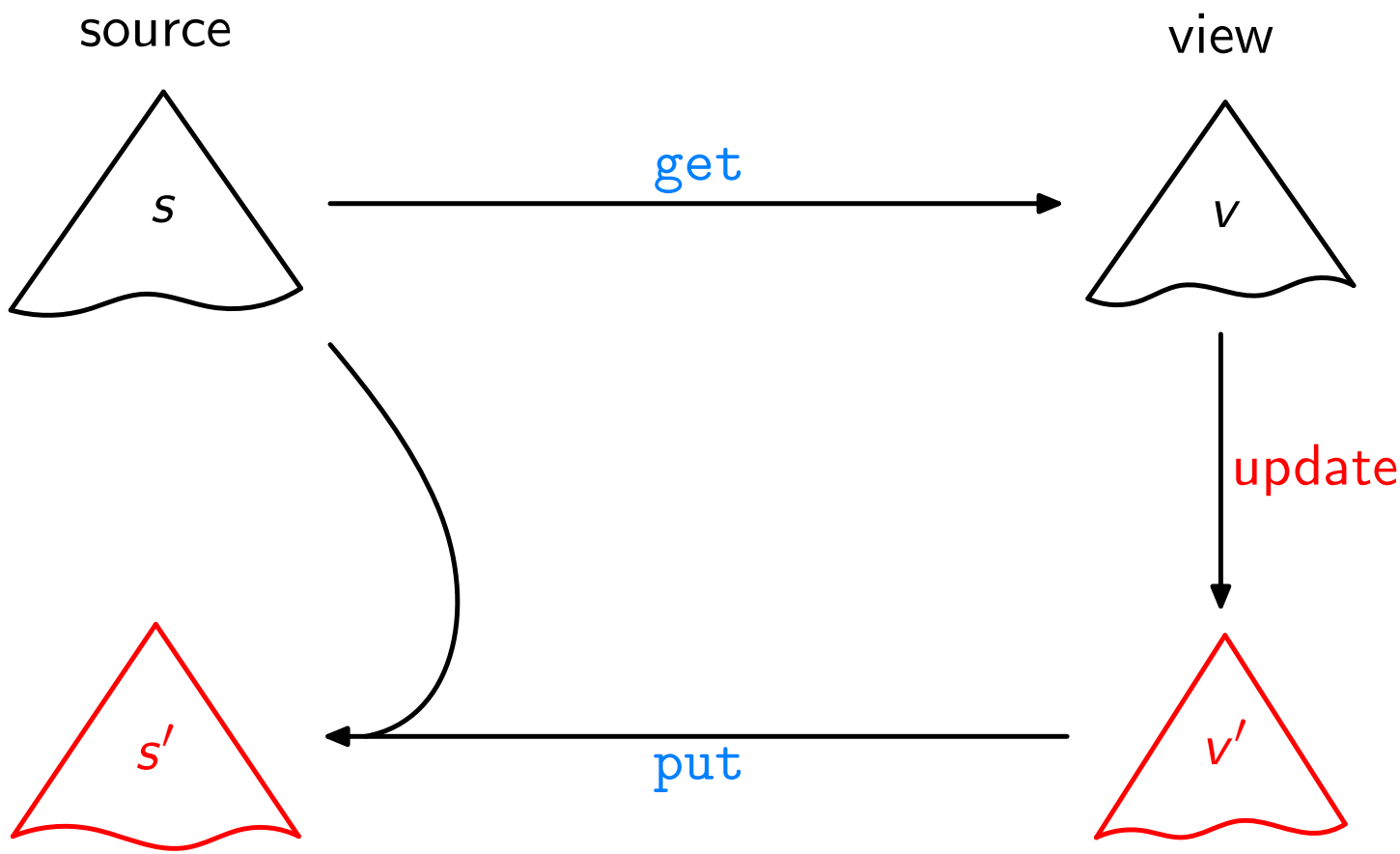

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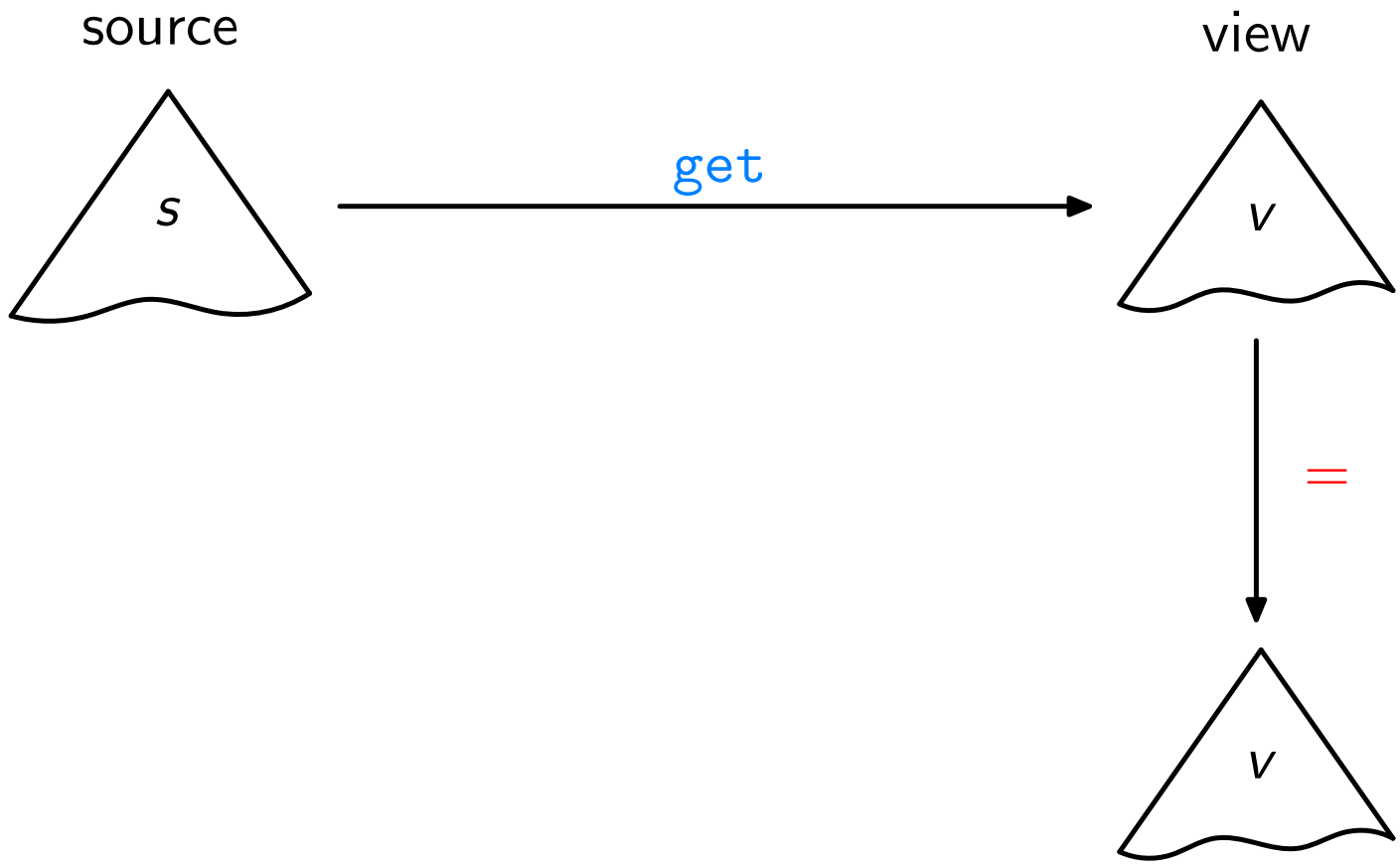
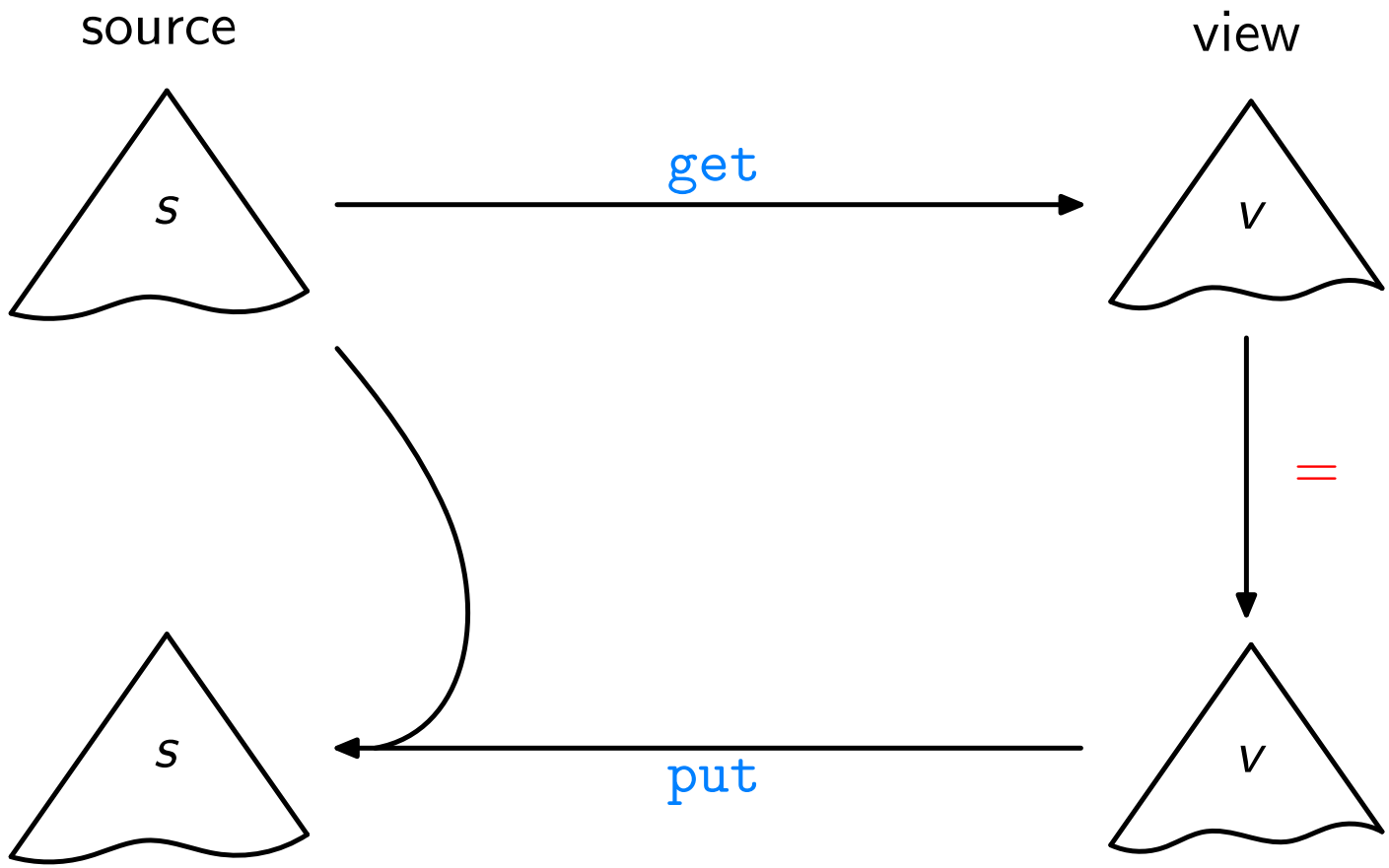


Acceptability / GetPut

Bidirectional Transformation



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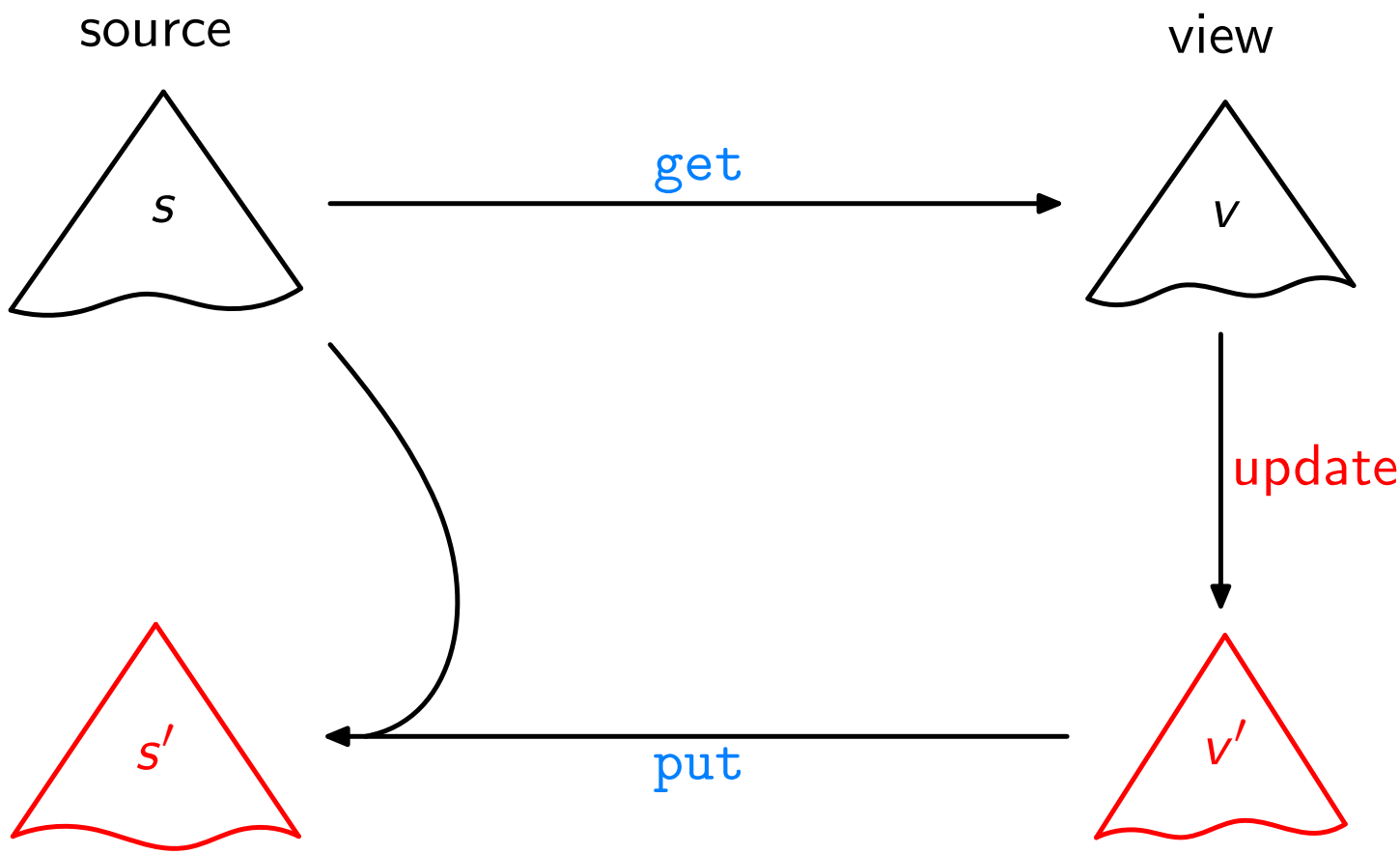

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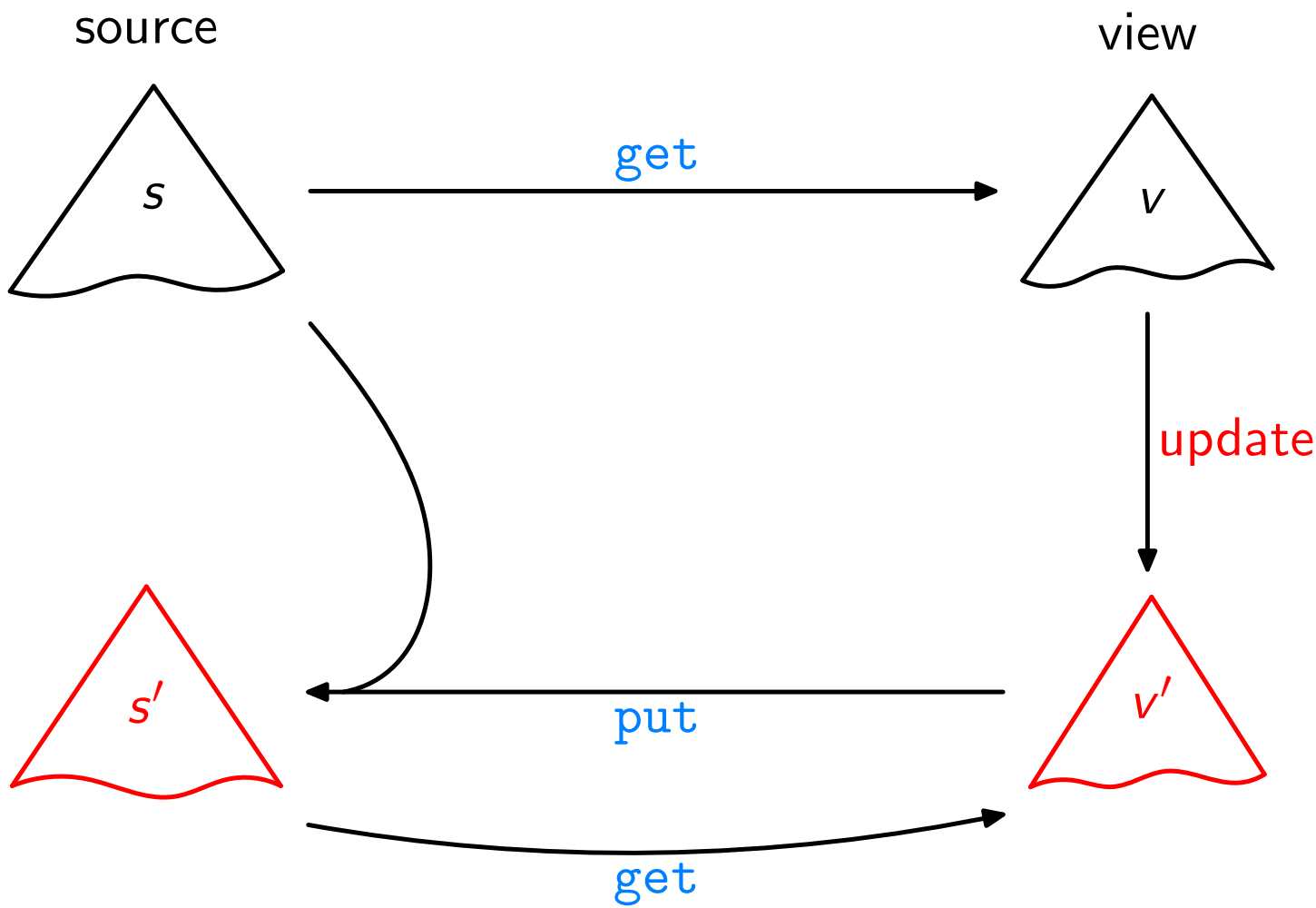

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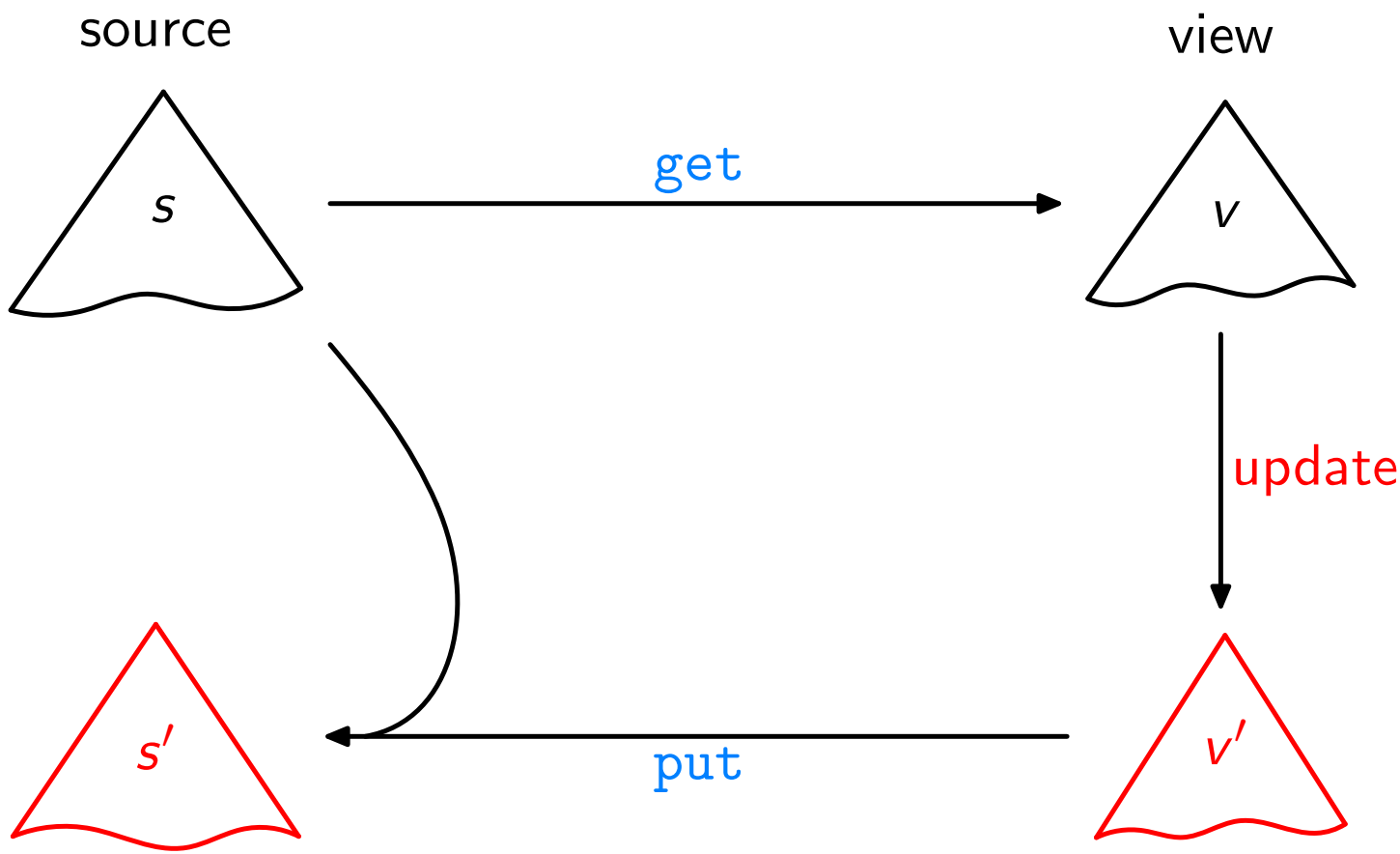

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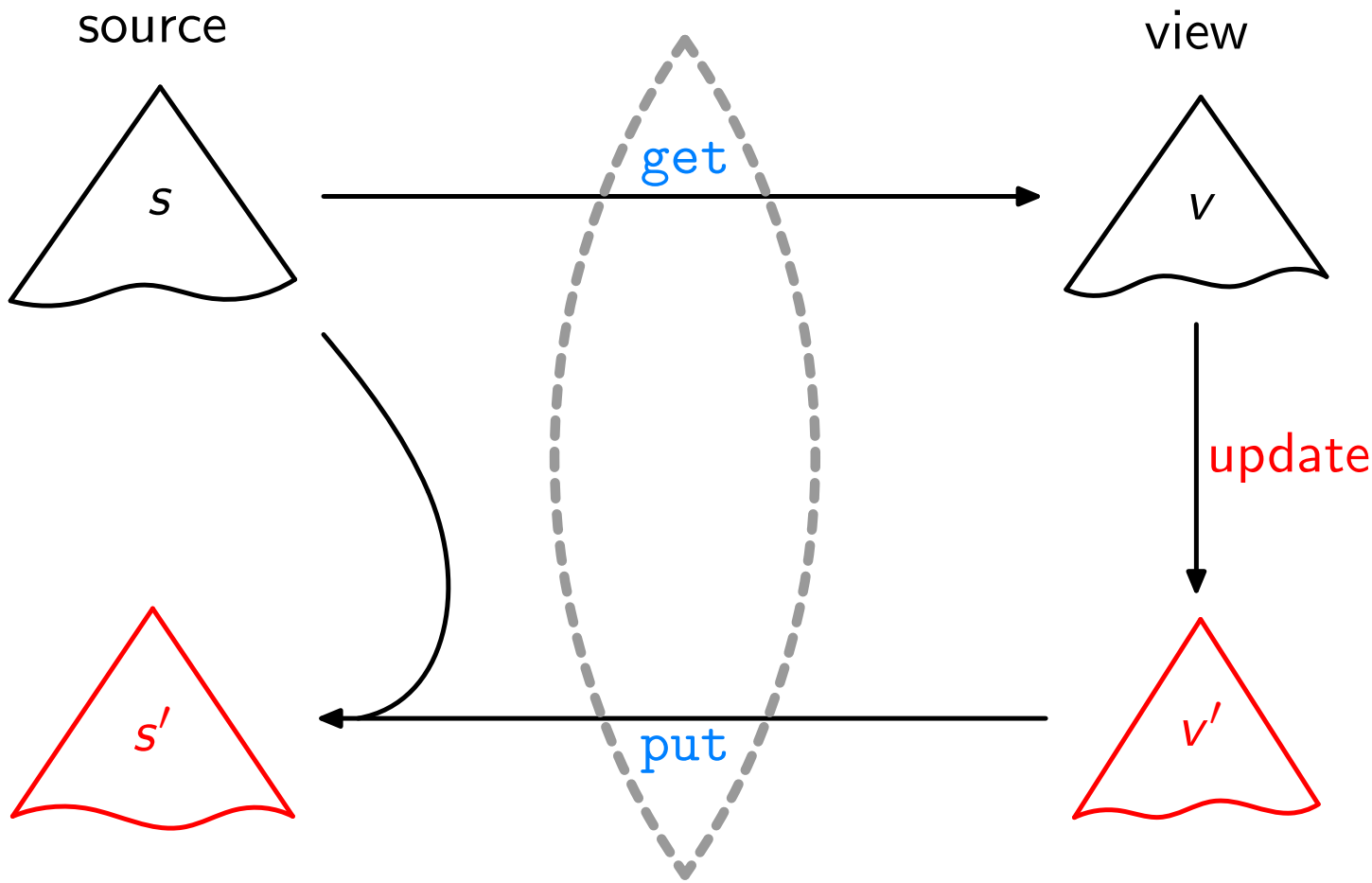
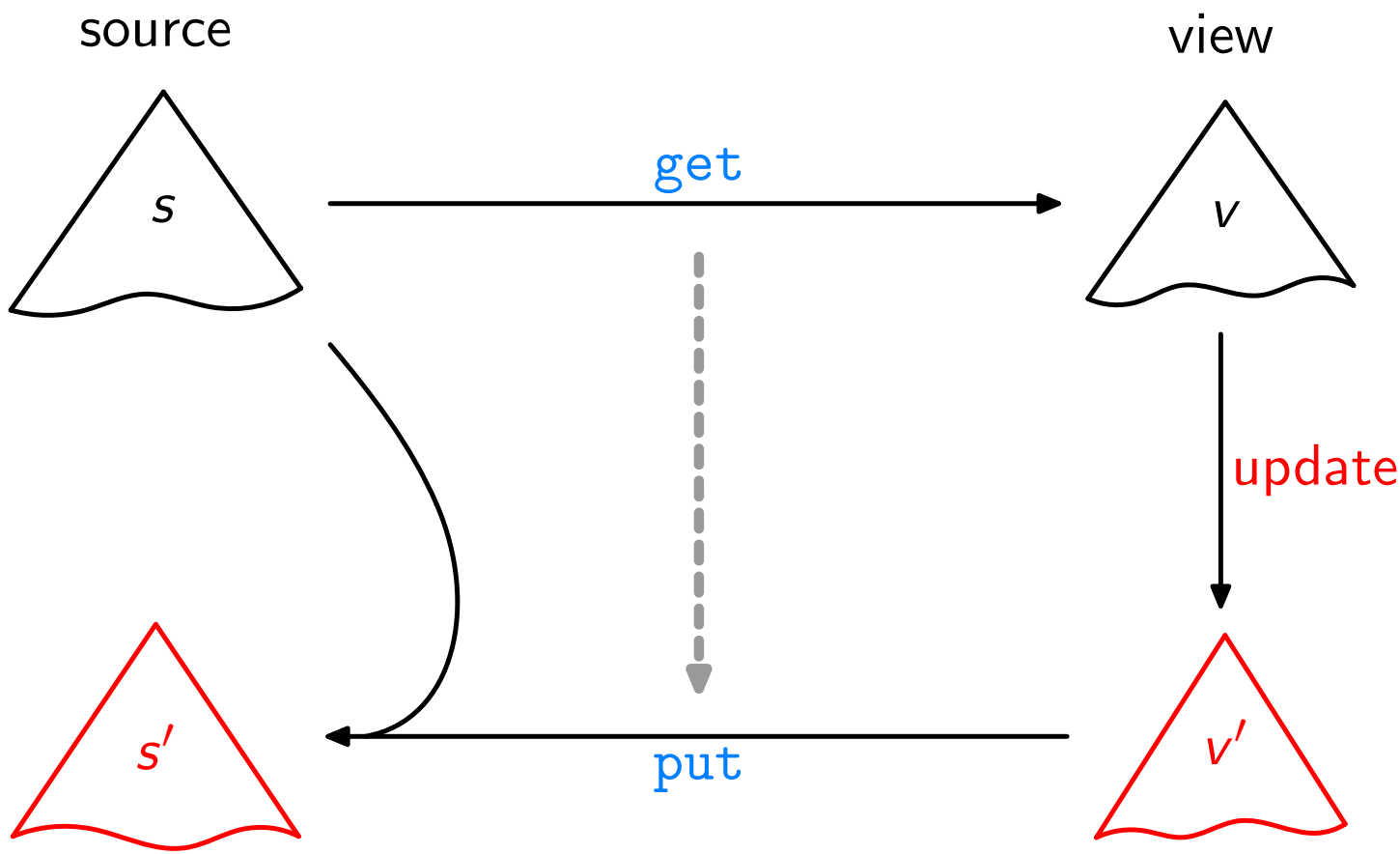


Lenses, DSLs

[Foster et al., TOPLAS'07, . . .]

Bidirectional Transformation



Bidirectionalization

[Matsuda et al., ICFP'07]

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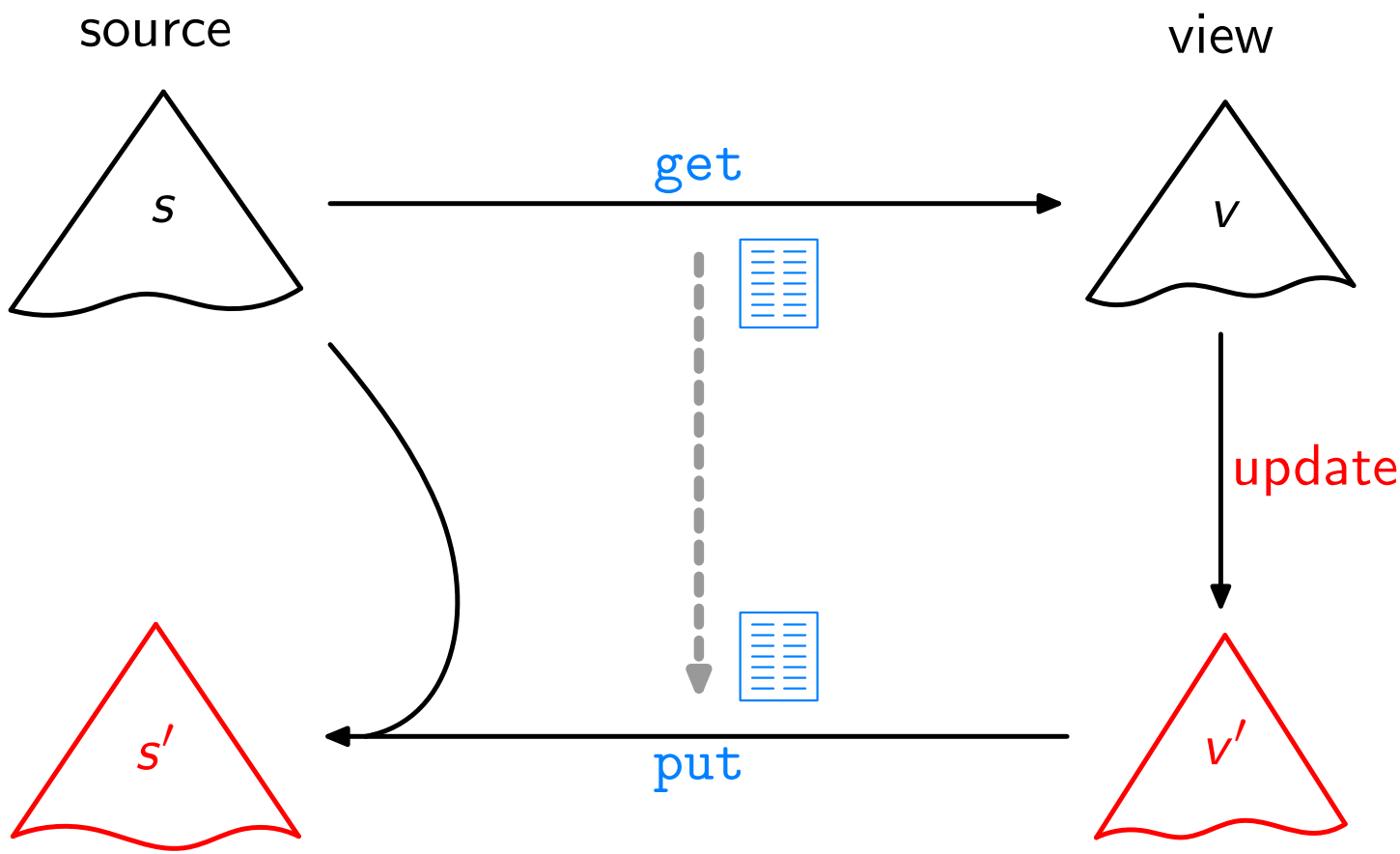

(Syntactic) Bidirectionalization

[Matsuda et al., ICFP'07]

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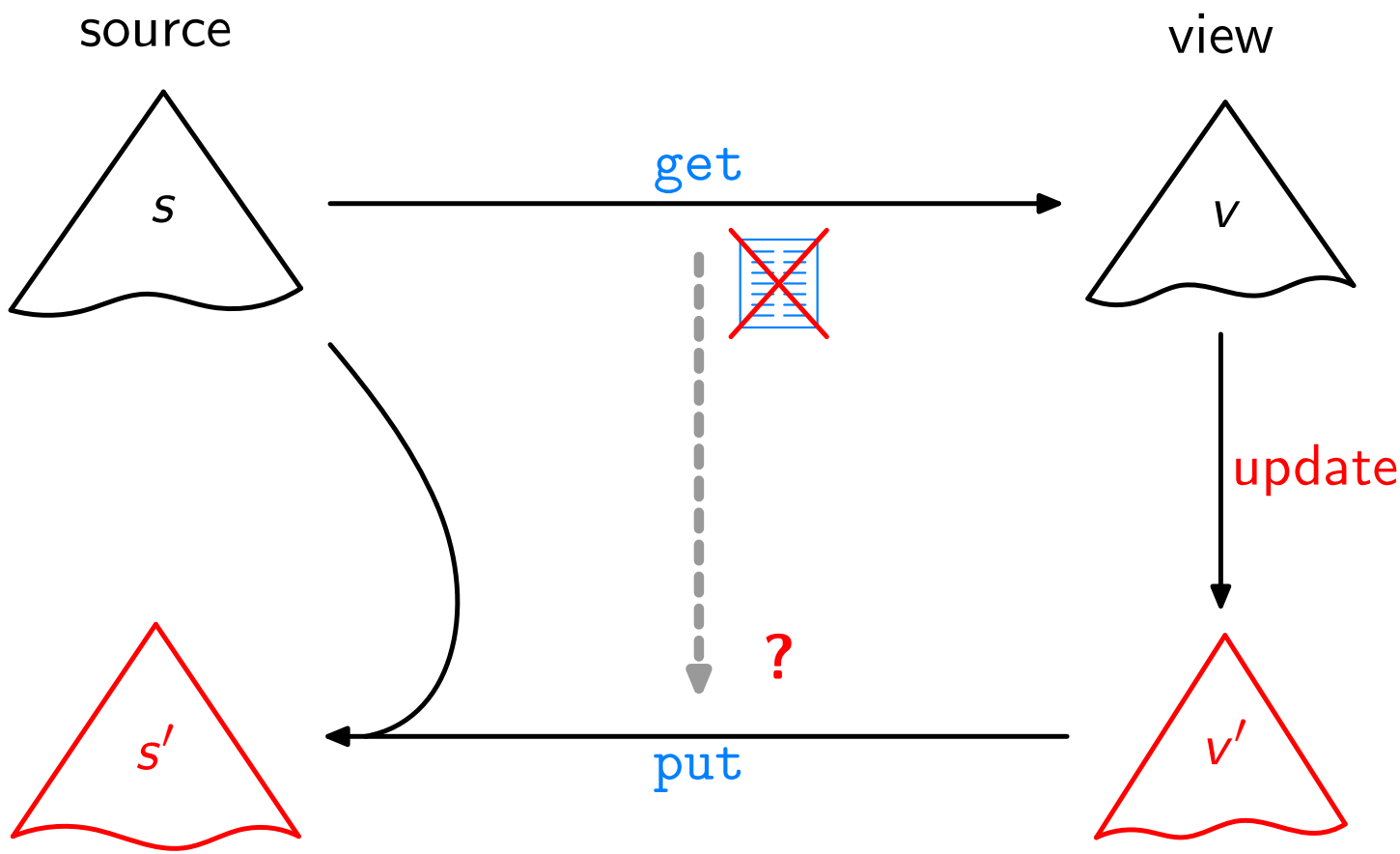

This work

Semantic Bidirectionalization

Aim: Write a higher-order function `bff` such that any `get` and `bff get` satisfy GetPut, PutGet, ...

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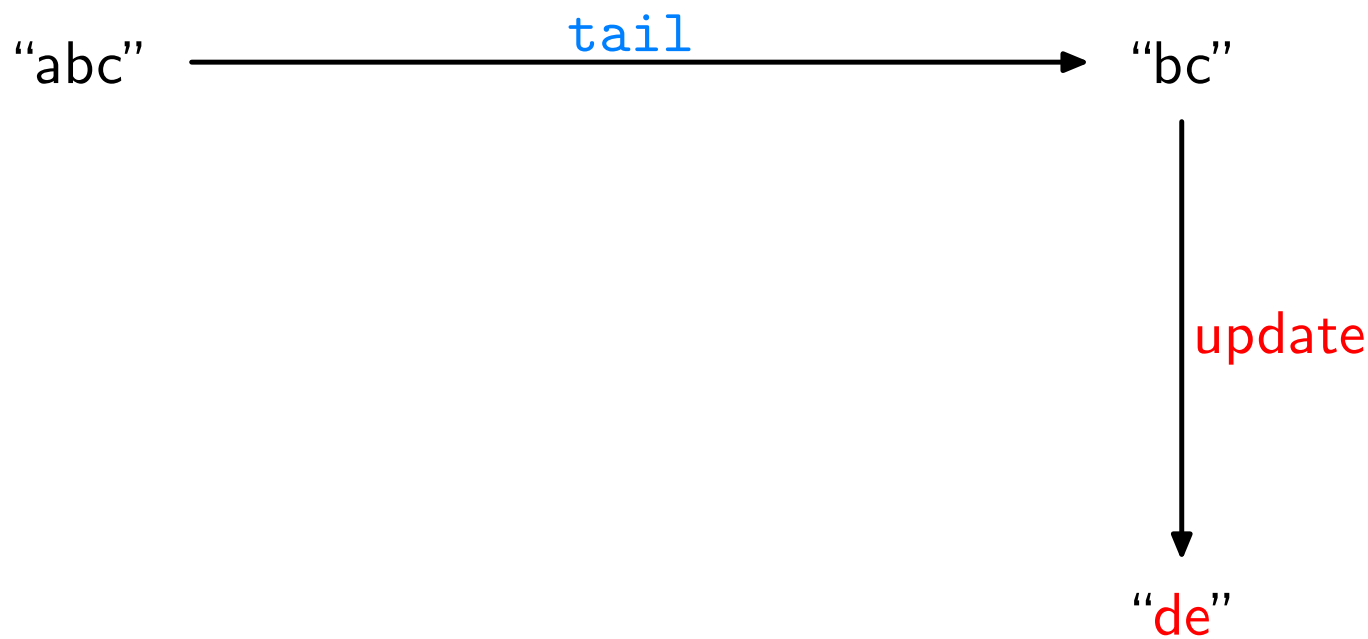

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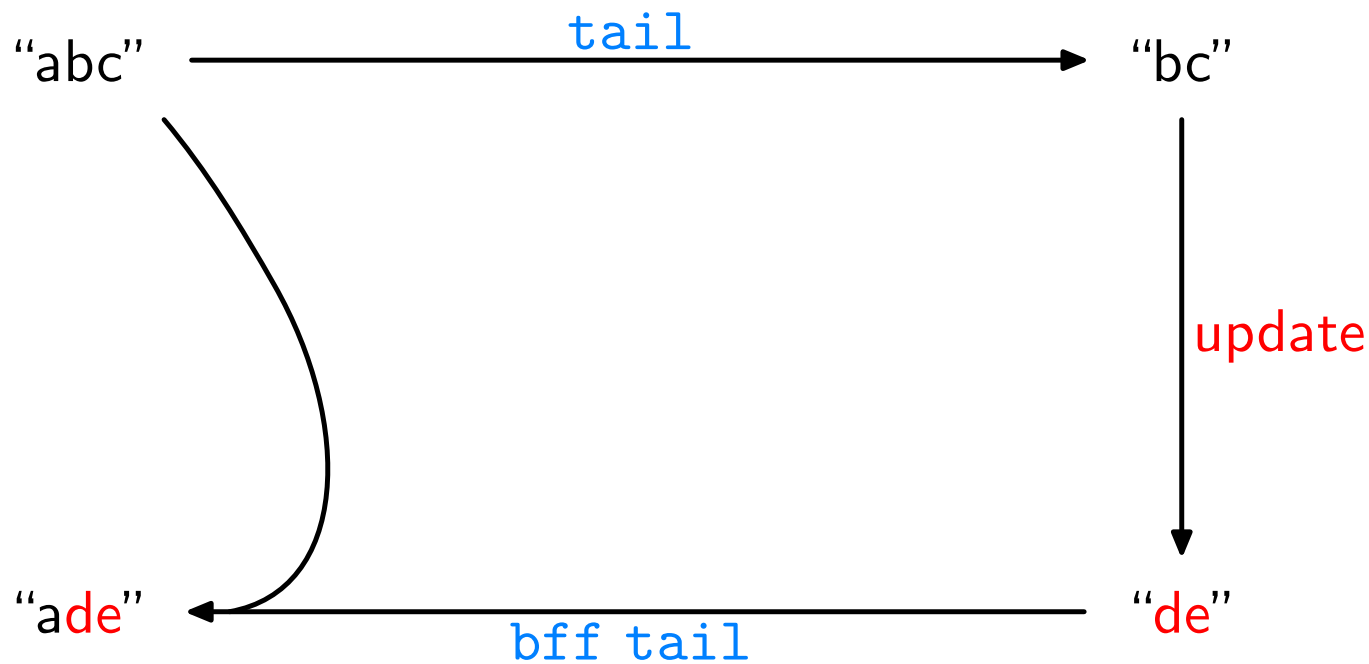

Analyzing Specific Instances

Assume we are given some

$$\texttt{get} :: [\alpha] \to [\alpha]$$

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Like:

$$\texttt{get}\ [0..n] = \begin{cases} [1..n] & \text{if } \texttt{get} = \texttt{tail} \\ [n..0] & \text{if } \texttt{get} = \texttt{reverse} \\ [0..(\texttt{min}\ 4\ n)] & \text{if } \texttt{get} = \texttt{take}\ 5 \\ \vdots & \end{cases}$$

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Then transfer the gained insights to source lists other than $[0..n]$!

Enter Free Theorems [Wadler, FPCA'89]

For every

$$\texttt{get} :: [\alpha] \to [\alpha]$$

we have

$$\texttt{map}\ f\ (\texttt{get}\ l) \quad = \quad \texttt{get}\ (\texttt{map}\ f\ l)$$

for arbitrary f and l, where

$$\begin{aligned}
&\texttt{map} :: (\alpha \to \beta) \to [\alpha] \to [\beta] \\
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Given an arbitrary list s of length $n+1$, set $f = (s\,!!)$, $l = [0..n]$, leading to:

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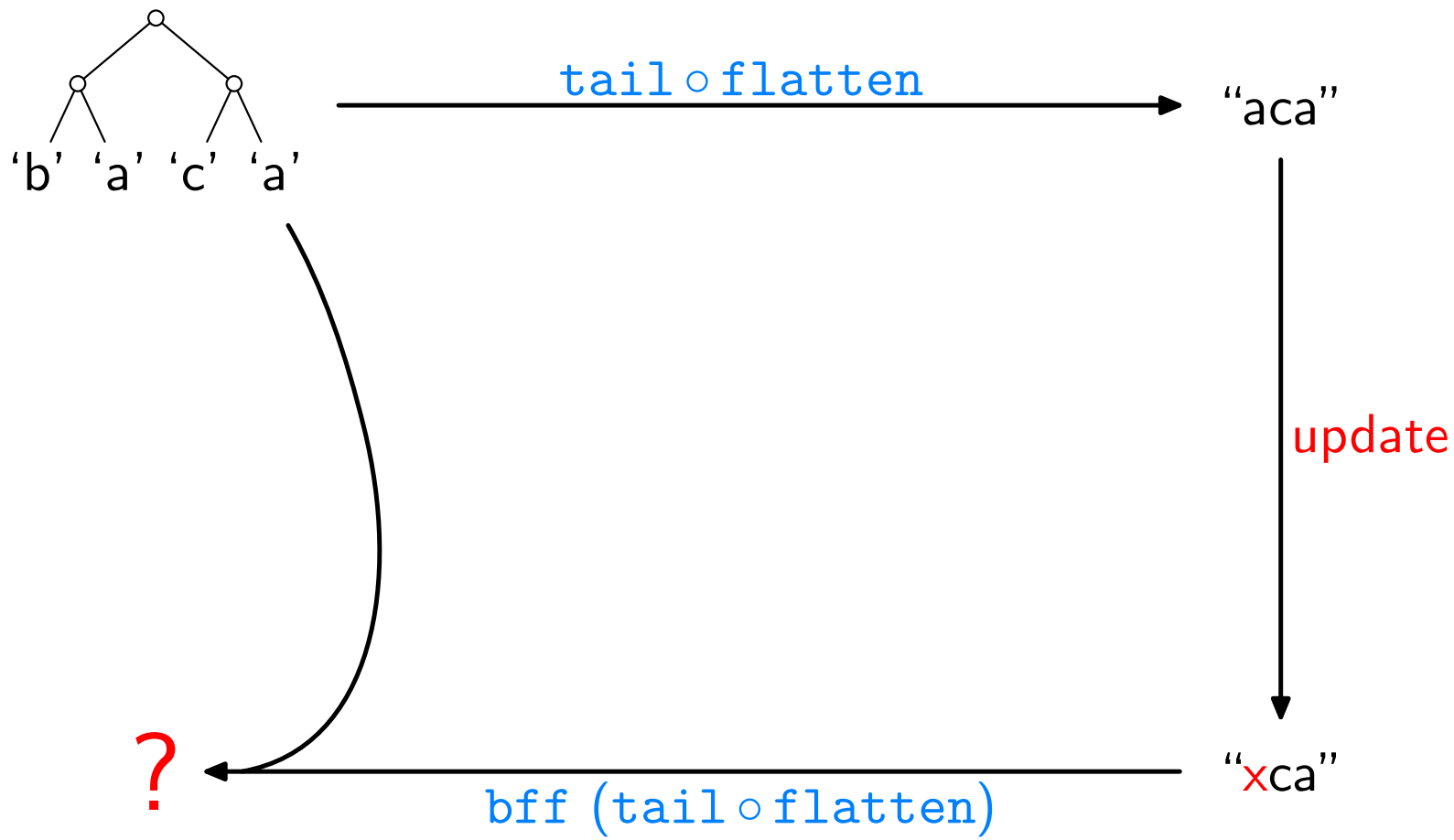
The Resulting Bidirectionalization Scheme by Example



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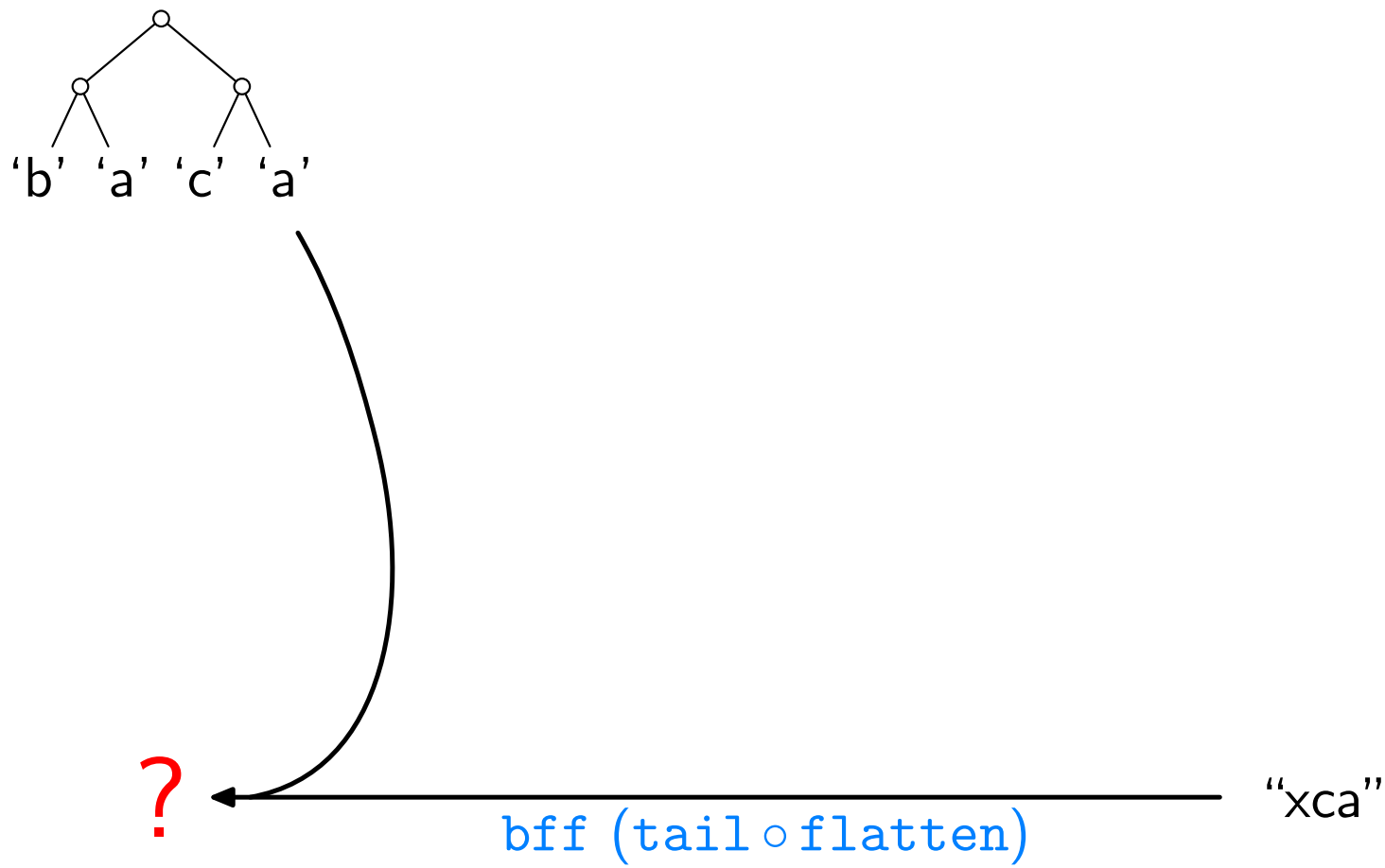

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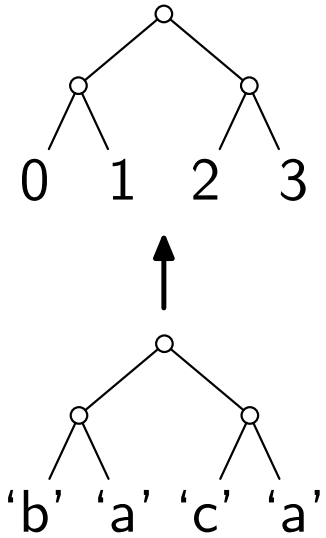

“xca”

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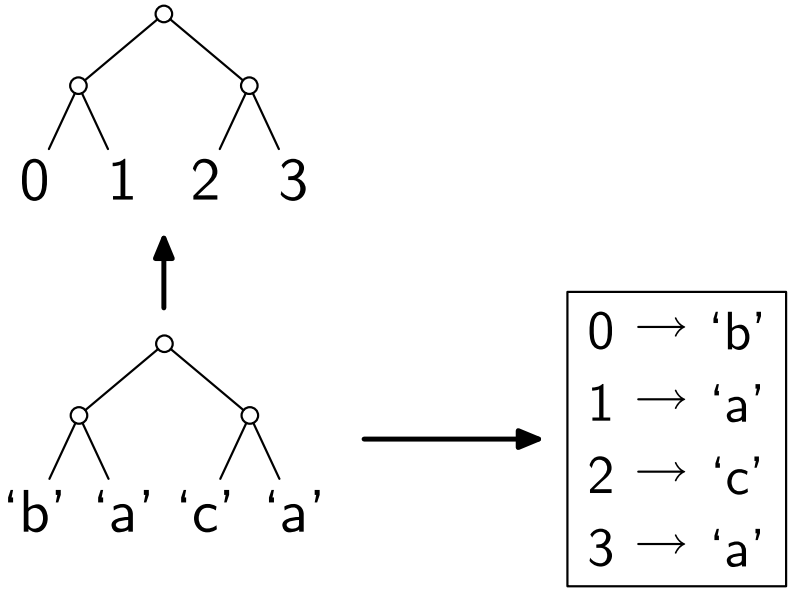

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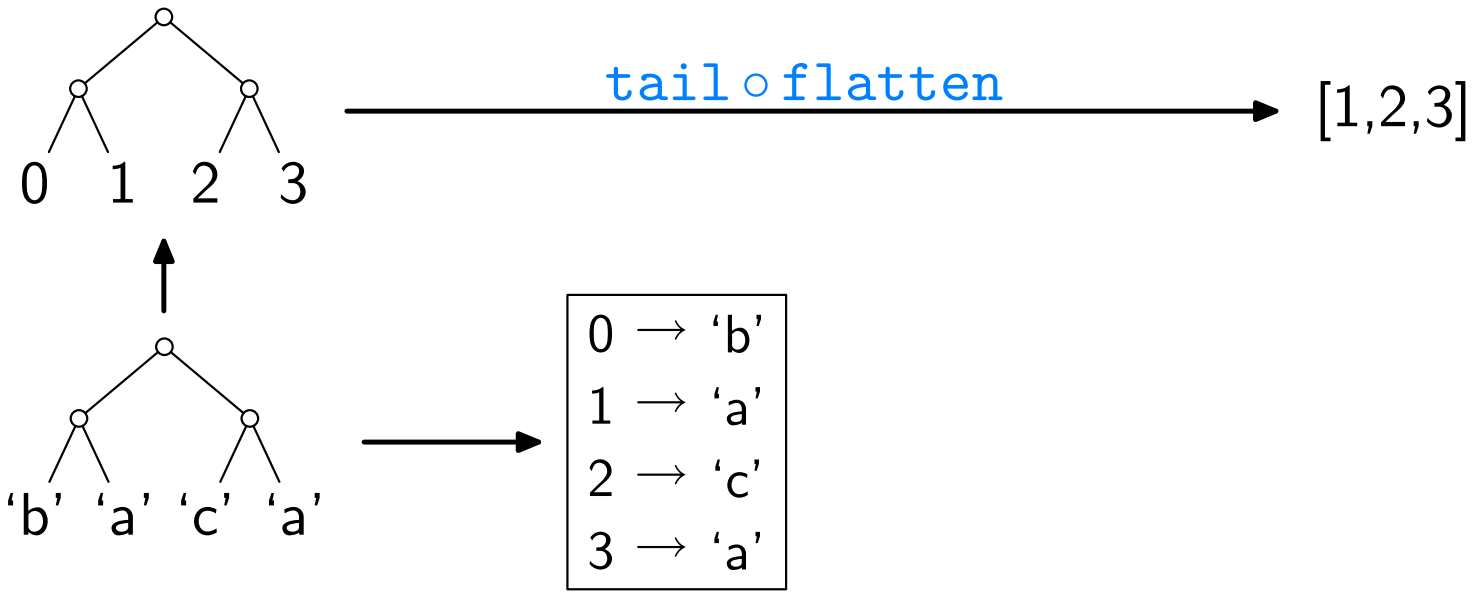

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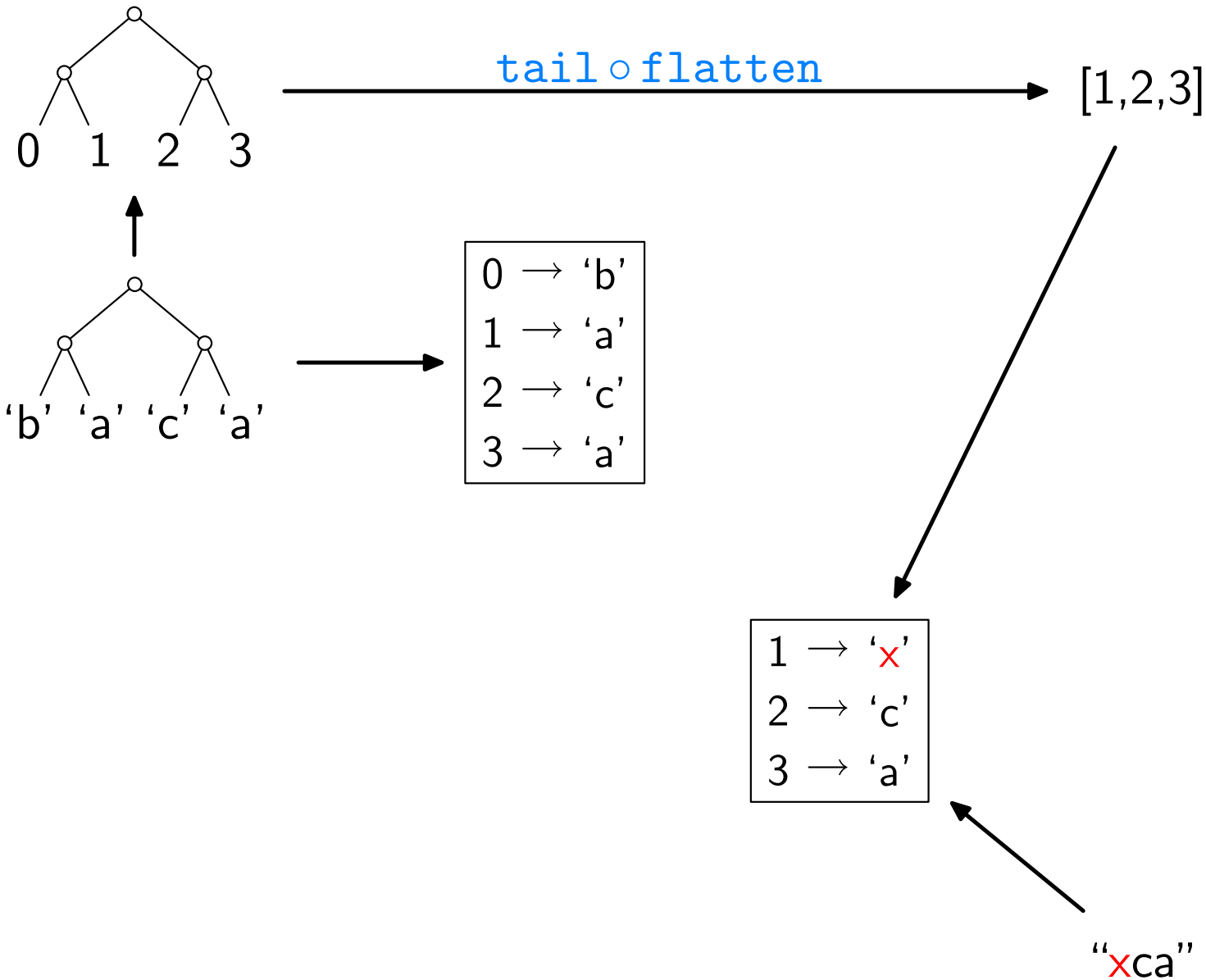

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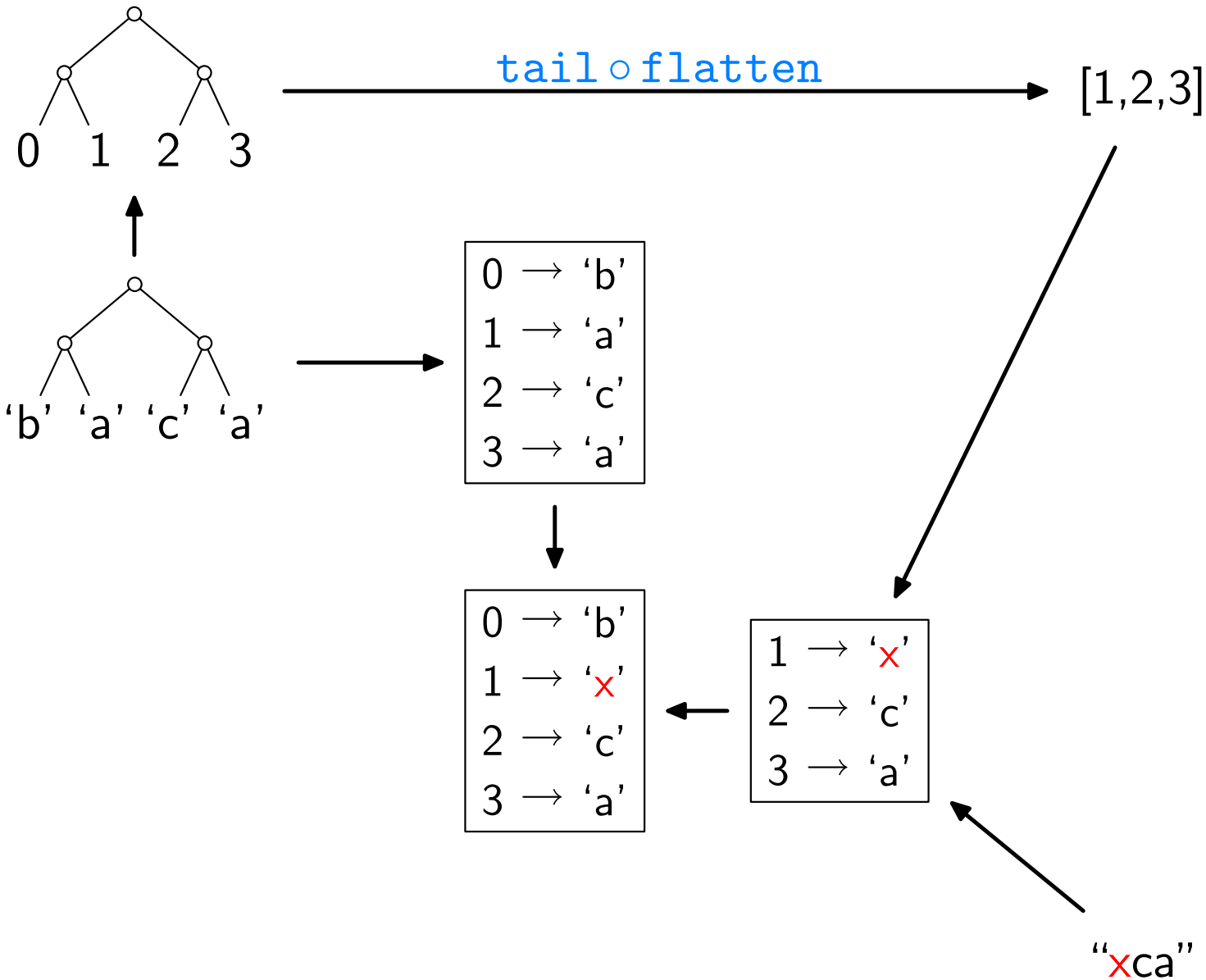

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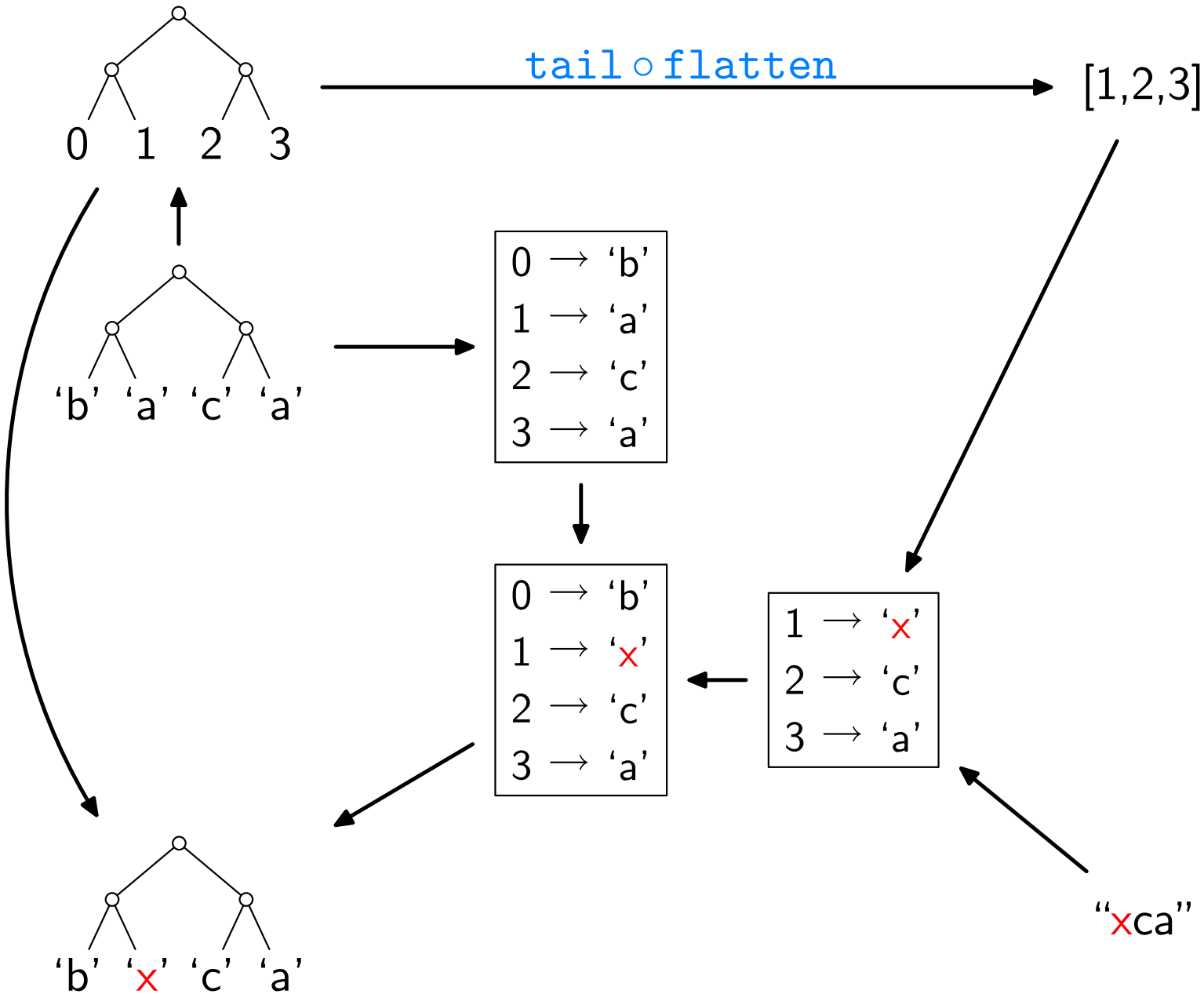

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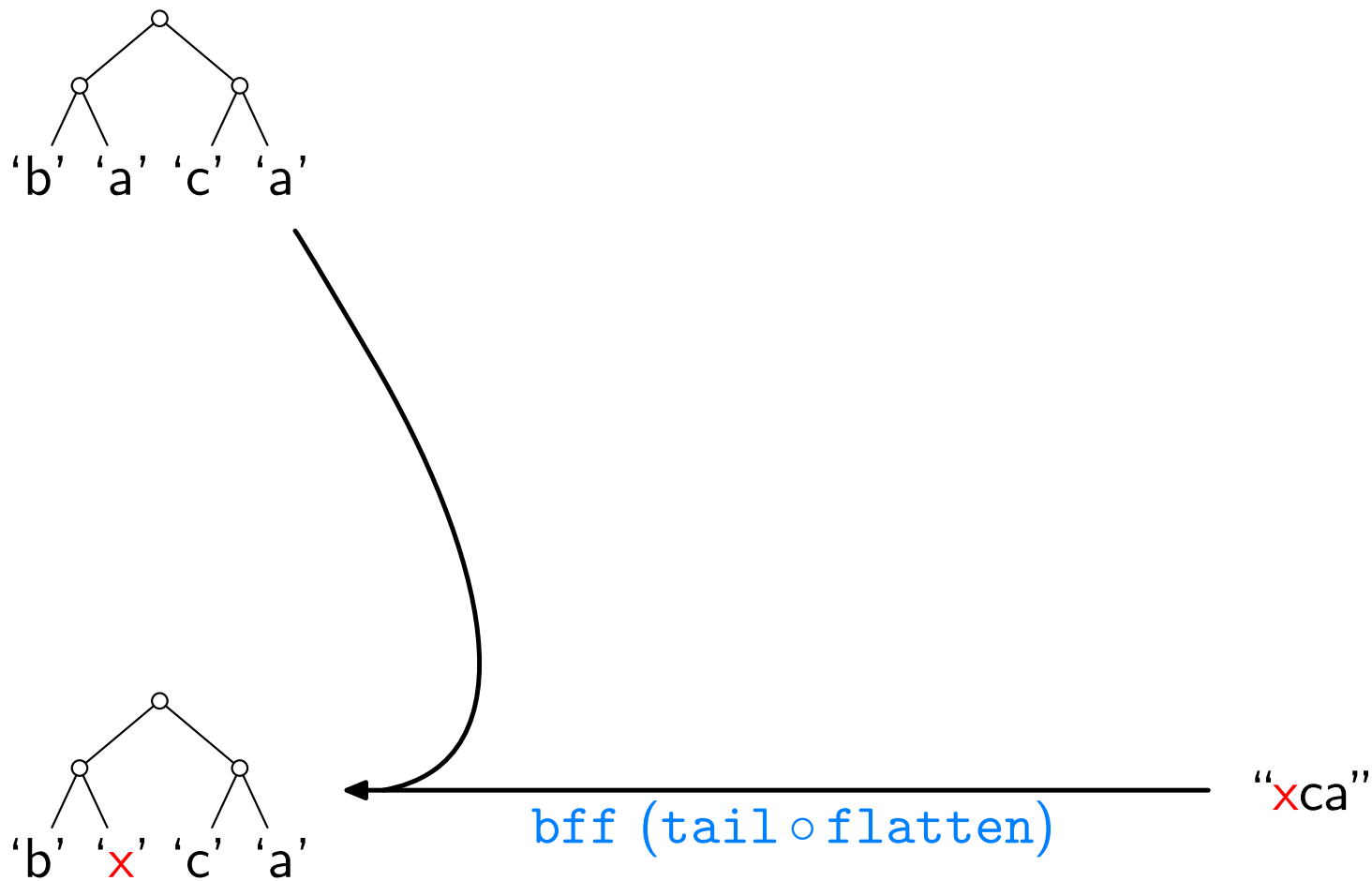

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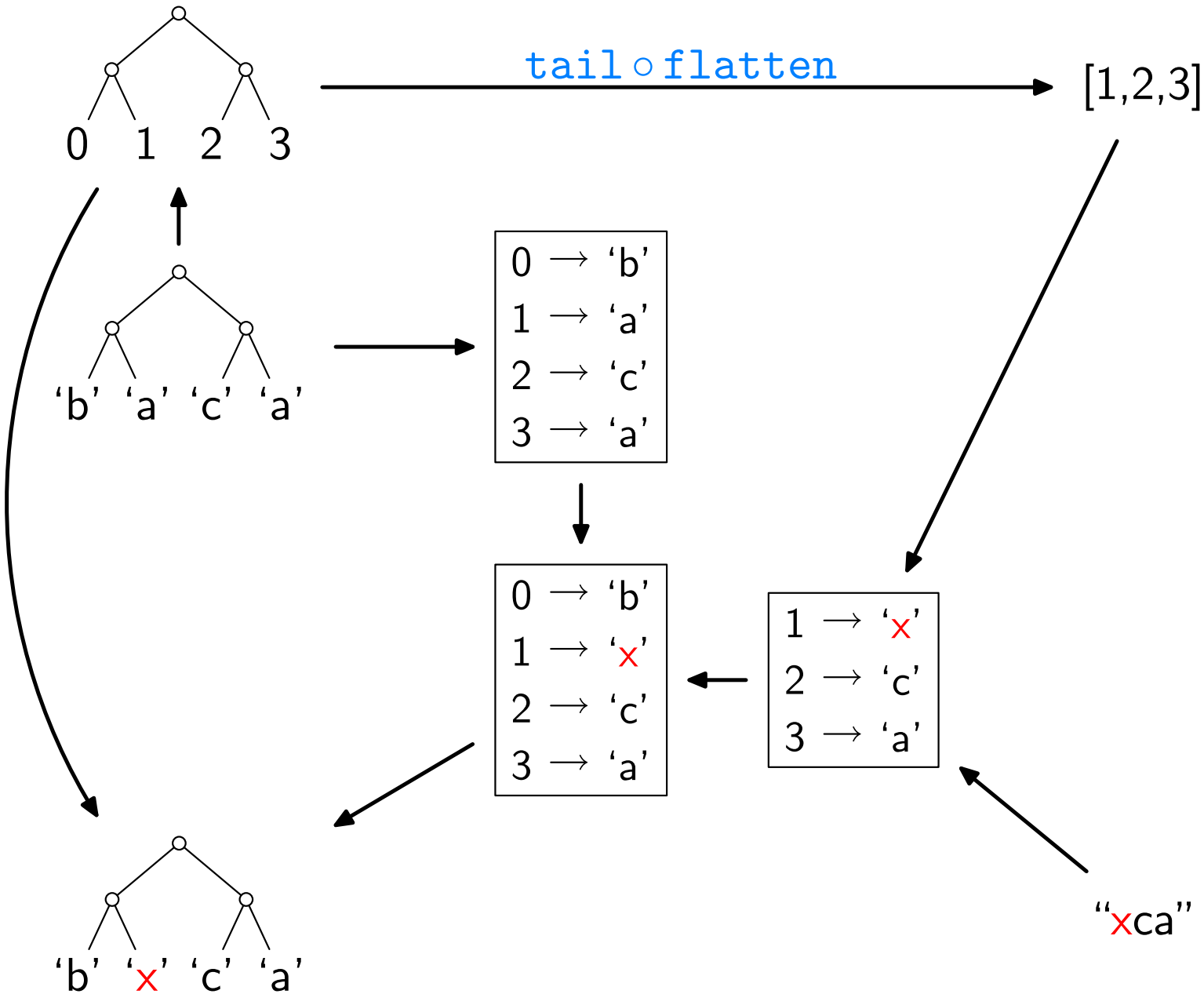

The Implementation (here: lists only, inefficient version)

$$
\begin{aligned}
\texttt{bff}\ \mathit{get}\ s\ v' = \textbf{let}\ n &= (\texttt{length}\ s) - 1 \\
t &= [0..n] \\
g &= \texttt{zip}\ t\ s \\
h &= \texttt{assoc}\ (\mathit{get}\ t)\ v' \\
h' &= h +\!\!+ g \\
\textbf{in}\ &\ \texttt{seq}\ h\ (\texttt{map}\ (\lambda i \to \texttt{fromJust}\ (\texttt{lookup}\ i\ h'))\ t)
\end{aligned}
$$

$$
\begin{aligned}
\texttt{assoc}\ [\,]\ \ \ [\,] &= [\,] \\
\texttt{assoc}\ (i : is)\ (b : bs) &= \textbf{let}\ m = \texttt{assoc}\ is\ bs \\
&\quad \textbf{in}\ \ \textbf{case}\ \texttt{lookup}\ i\ m\ \textbf{of} \\
&\qquad \text{Nothing} \to (i, b) : m \\
&\qquad \text{Just}\ c \mid b == c \to m
\end{aligned}
$$

The Implementation (here: lists only, inefficient version)

```
bff get s v' = let n = (length s) - 1
                   t = [0..n]
                   g = zip t s
                   h = assoc (get t) v'
                   h' = h ++ g
               in  seq h (map (λi → fromJust (lookup i h')) t)

assoc []       []       = []
assoc (i : is) (b : bs) = let m = assoc is bs
                          in  case lookup i m of
                                Nothing          → (i, b) : m
                                Just c | b == c  → m
```

- for the actual (slightly more elaborate) code, see the paper
- try out: `http://linux.tcs.inf.tu-dresden.de/~bff`

What Else?

In the paper:

- treatment of equality and ordering constraints
- full proofs, using free theorems and equational reasoning
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Pros of the approach:

- great fun
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- very lightweight, easy access to bidirectionality

Cons of the approach:

- efficiency still leaves room for improvement
- partiality, e.g., rejection of shape-affecting updates so far

References

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Theorems for free!
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