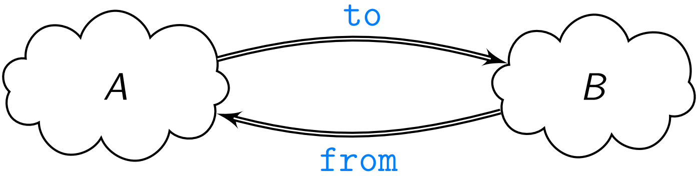
# Programming Language Approaches to Bidirectional Transformation

Janis Voigtländer

University of Bonn

LDTA'12

# Bidirectional Transformations (BX)



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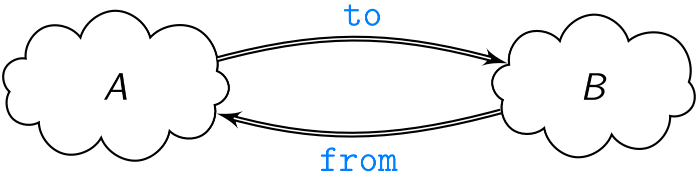


concrete syntax $\Leftrightarrow$ abstract syntax

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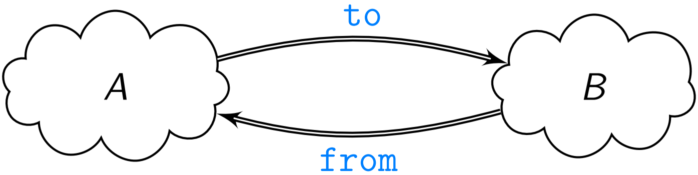


database source $\Leftrightarrow$ materialized view

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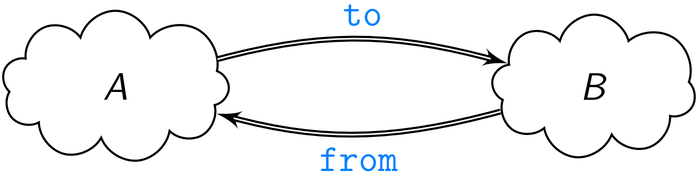


document representation $\Leftrightarrow$ screen visualization

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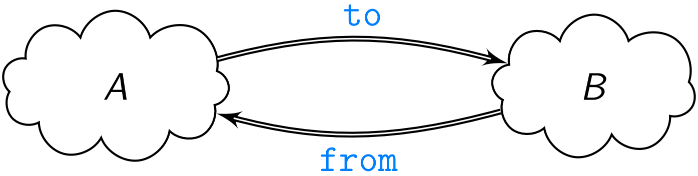


software model ⇔ code

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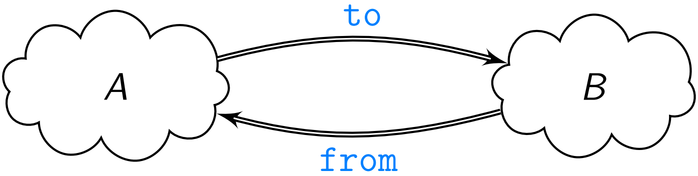


abstract datatype $\Leftrightarrow$ actual implementation

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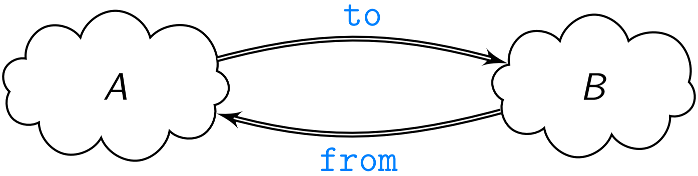


program input $\Leftrightarrow$ program output

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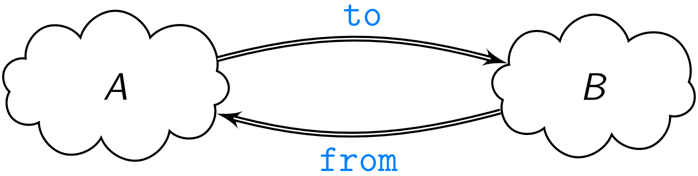


| | | |
|---|---|---|
| concrete syntax | $\Leftrightarrow$ | abstract syntax |
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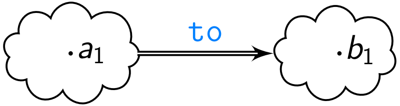

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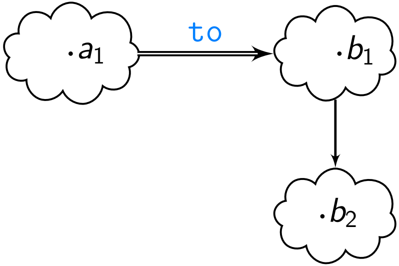

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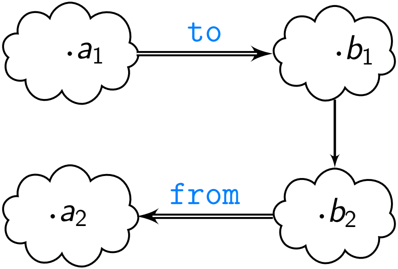

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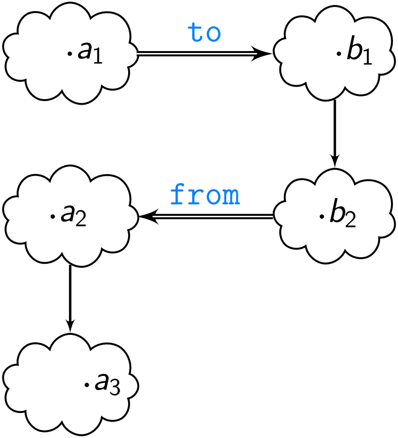

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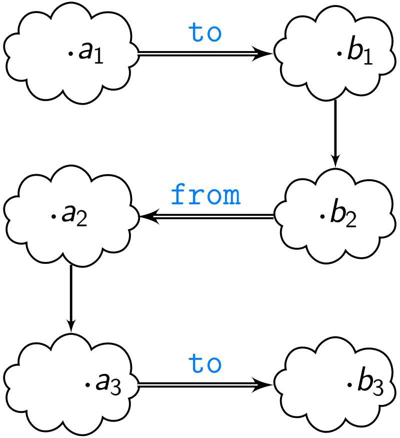

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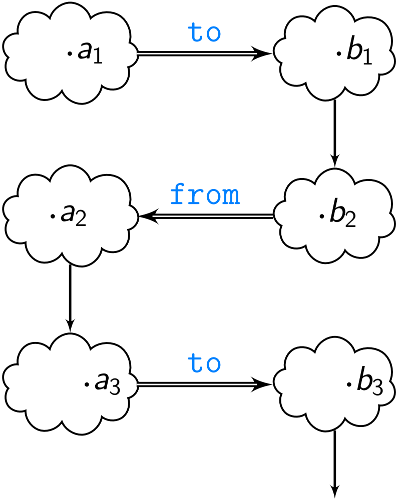

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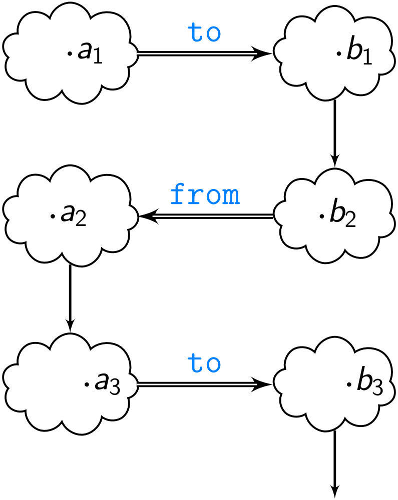


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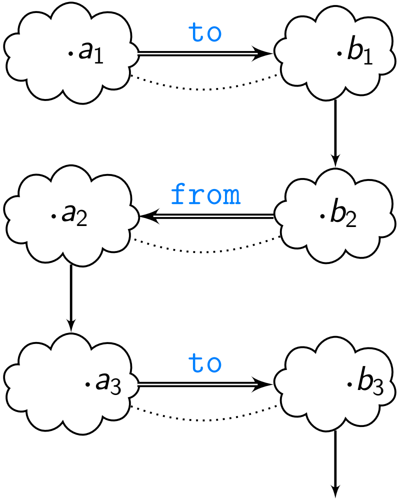


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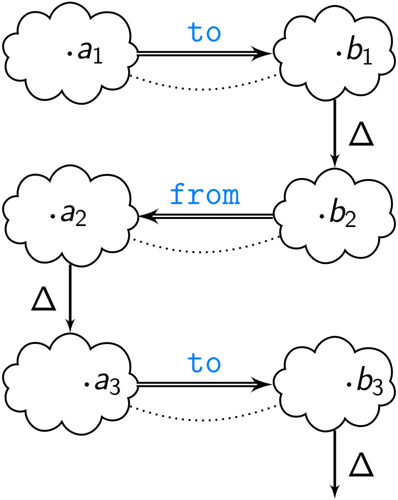


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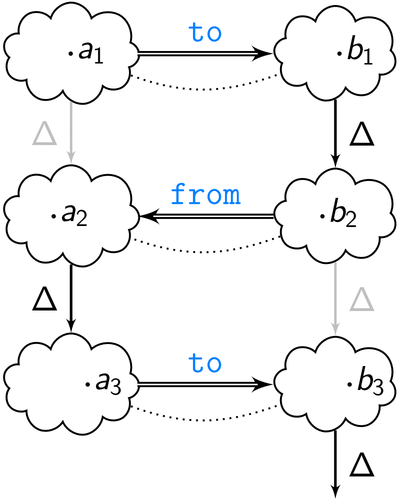


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# Bidirectional Transformations

A closer look at representing $a_i$ $b_i$ connections.

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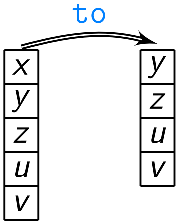

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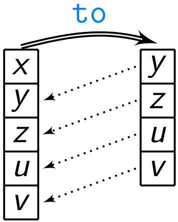

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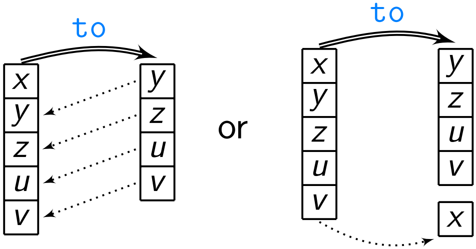

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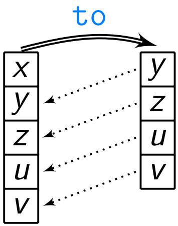


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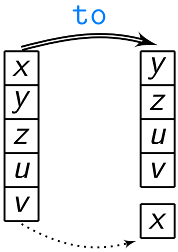


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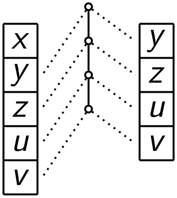

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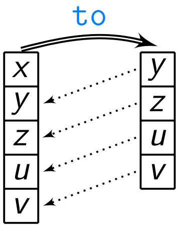


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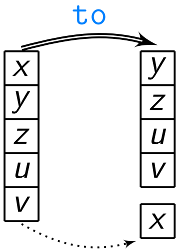


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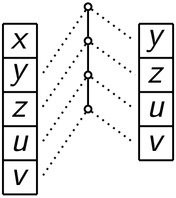


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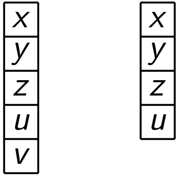

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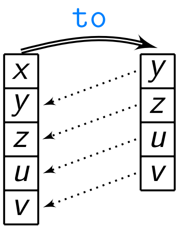


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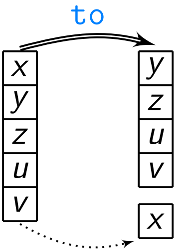


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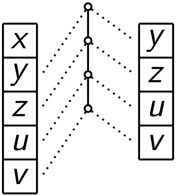


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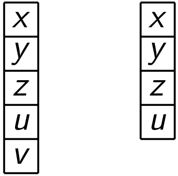


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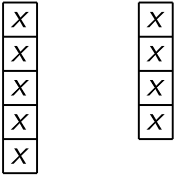

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Some further relevant aspects:

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answers/approaches vary with field

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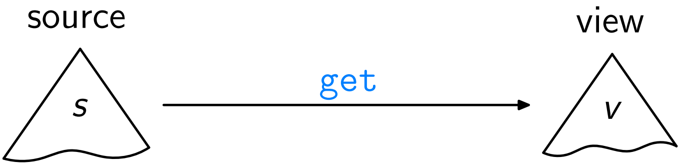
A specific (asymmetric) setting:



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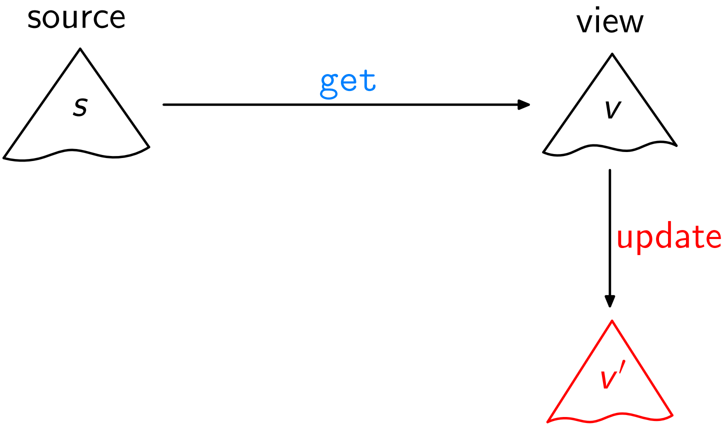

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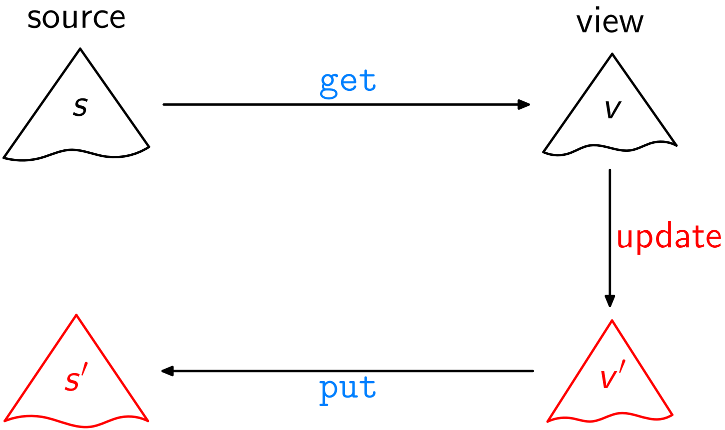

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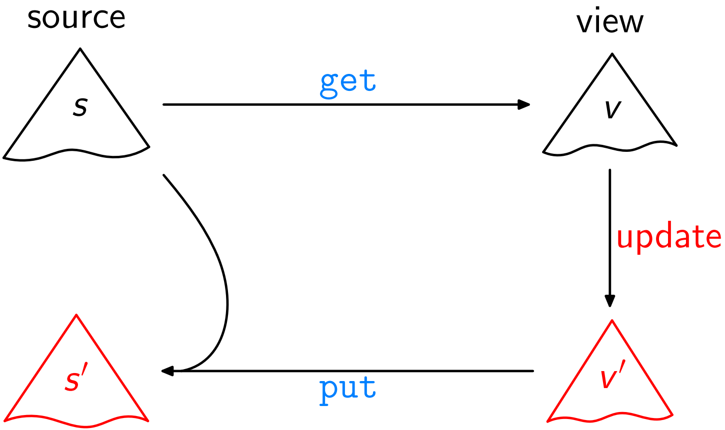

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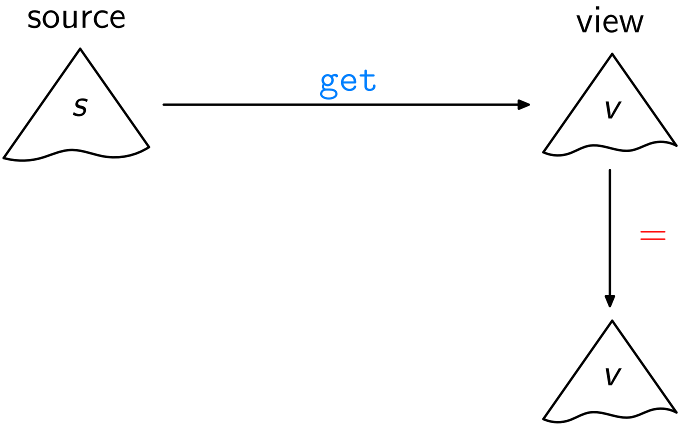


GetPut law

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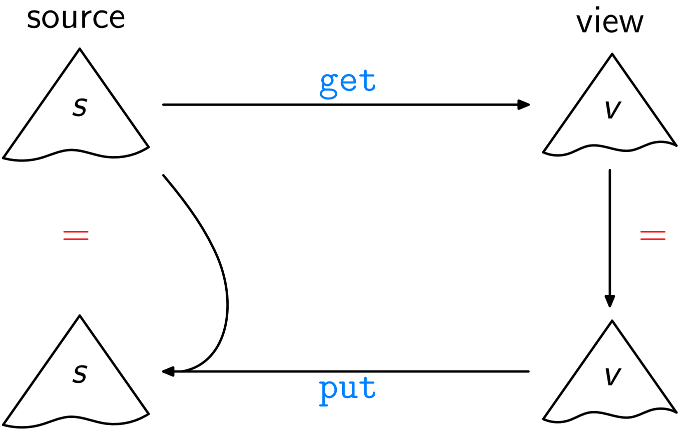


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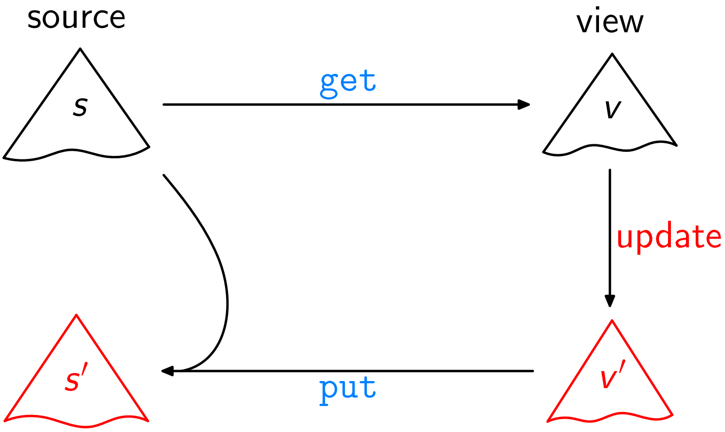


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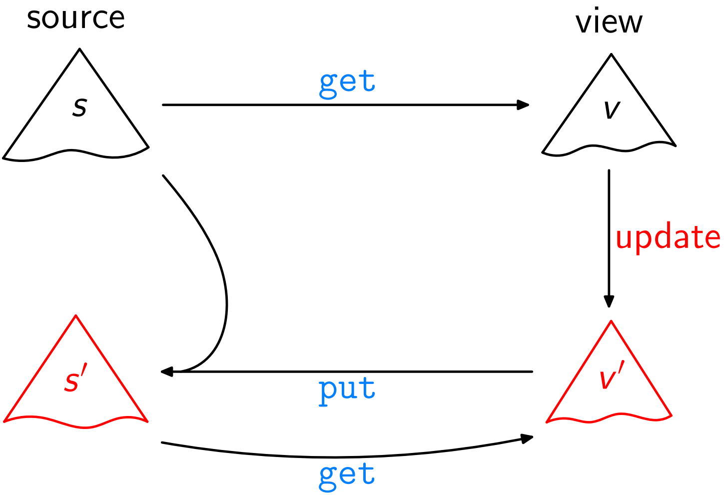


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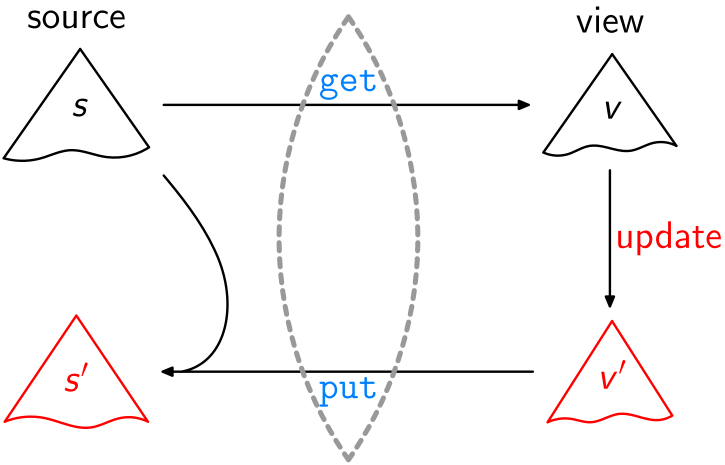

## Bidirectionalization “by Hand”

A simple example:

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&\texttt{get} :: [\alpha] \to [\alpha] \\
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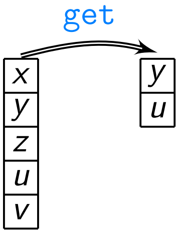

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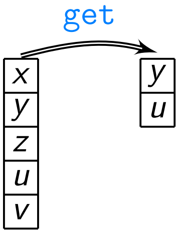


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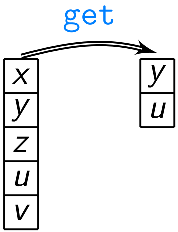


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There has been, and is ongoing, great work in the "lenses" PL/DSLs tradition [Foster et al., ACM TOPLAS'07, …]. Not covered today.

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We will mention/look at:

- syntactic program transformation
- semantic/type-based transformation
- benefits of higher-order types and abstraction
- search-based program synthesis (if time permits, otherwise see PEPM'12 short paper)

# A Principled Approach: Constant-Complement [Bancilhon & Spyratos, ACM TODS'81]

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has to be effective!

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other choices possible, and give different behavior

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# Automatic Bidirectionalization by Example

Let:

$$\begin{aligned}
&\mathtt{get} :: [\alpha] \to [\alpha] \\
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&\mathtt{get}\ (x : y : zs) && = y : (\mathtt{get}\ zs)
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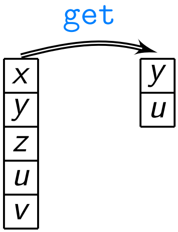

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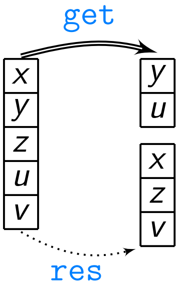


A syntactically derived complement function:

$$\begin{aligned}
&\texttt{res}\ [\,] &&= \mathsf{C}_1 \\
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Syntactic pairing:

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Syntactic inversion:

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corresponds to (the earlier seen):

$$
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&\texttt{put}\ [\,] && [\,] && = [\,] \\
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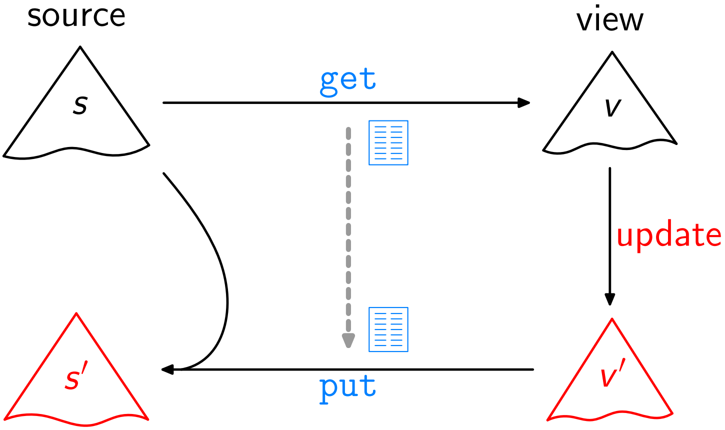


Syntactic Bidirectionalization

[Matsuda et al., ICFP'07]

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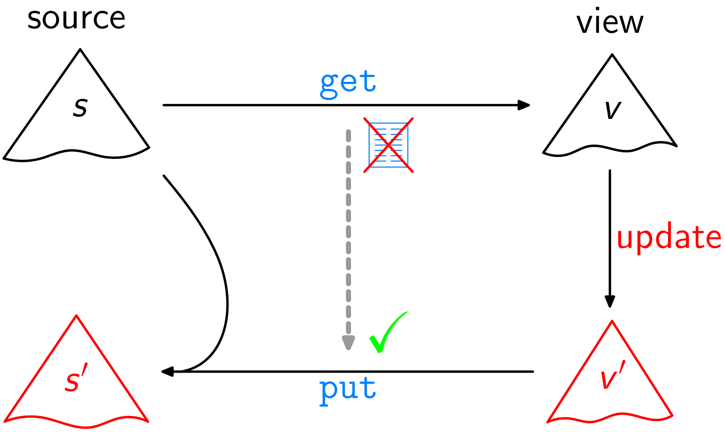


Semantic Bidirectionalization

[V., POPL'09]

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Aim: Write higher-order function `bff`[†] such that any `get` and `bff get` satisfy GetPut, PutGet, . . . .

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$$\text{“abc”} \xrightarrow{\texttt{tail}} \text{“bc”}$$

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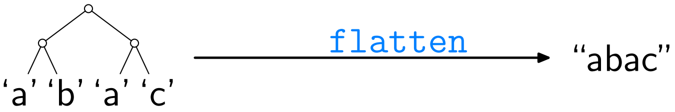


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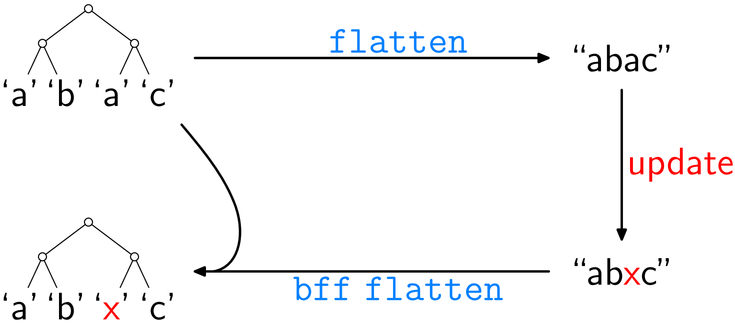


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Assume we are given some

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$$\texttt{get}\ [1..n] = \begin{cases} [2..n] & \text{if } \texttt{get} = \texttt{tail} \\ [n..1] & \text{if } \texttt{get} = \texttt{reverse} \\ [1..(\texttt{min}\ 5\ n)] & \text{if } \texttt{get} = \texttt{take}\ 5 \\ & \vdots \end{cases}$$

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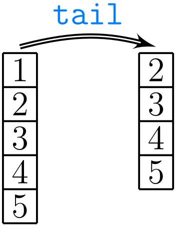

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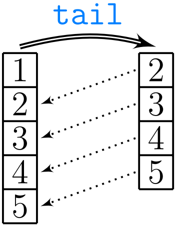

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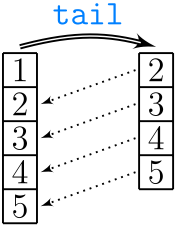


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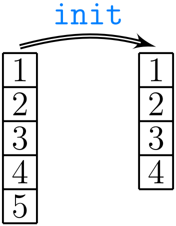

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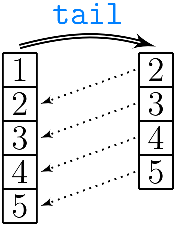


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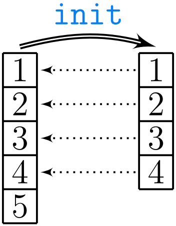

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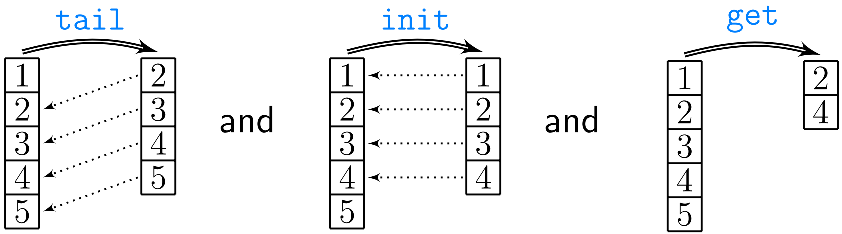

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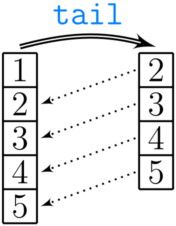


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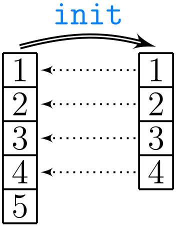


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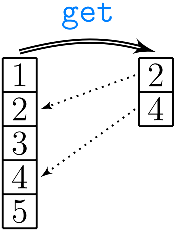

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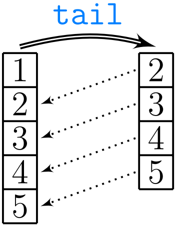


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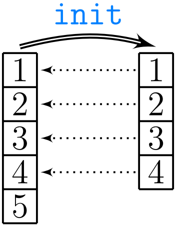


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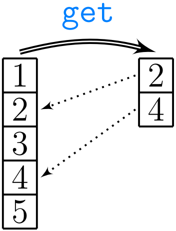


Then transfer the gained insights to arbitrary lists!

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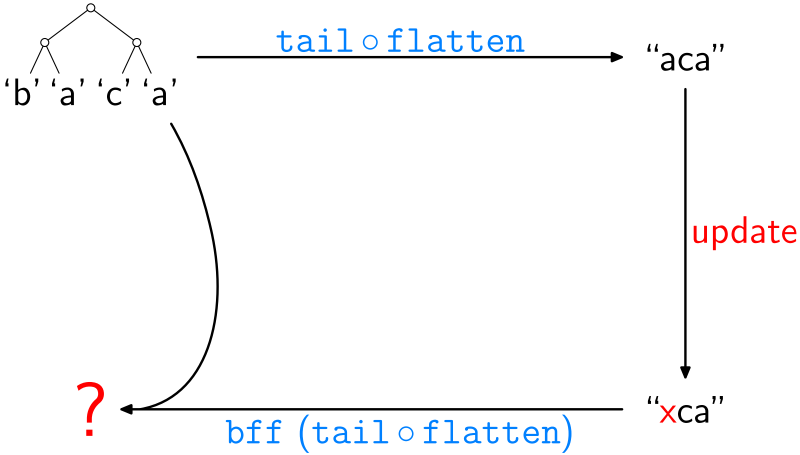

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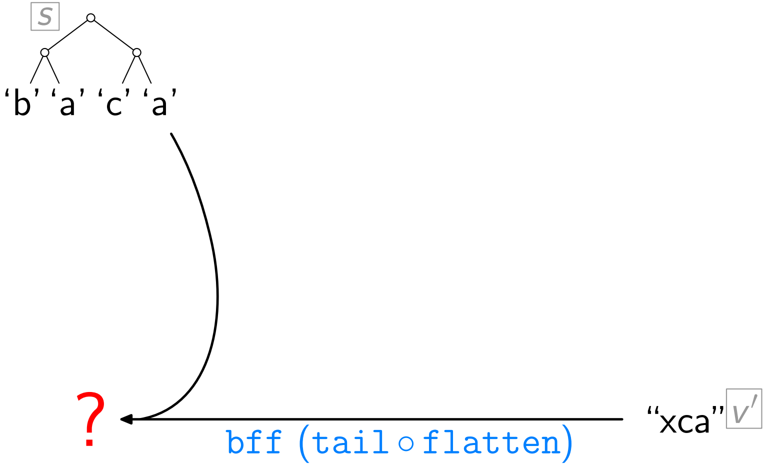

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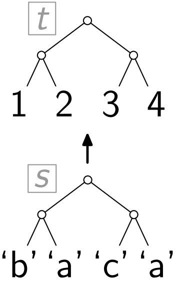

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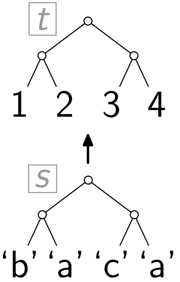


$1 \to$ 'b'
$2 \to$ 'a'
$3 \to$ 'c'
$4 \to$ 'a'

"xca" $v'$

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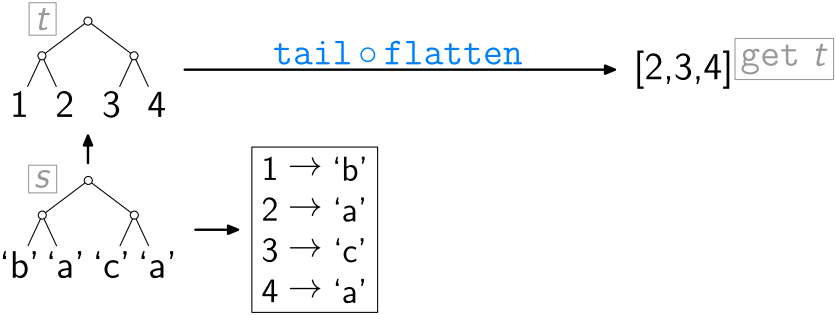


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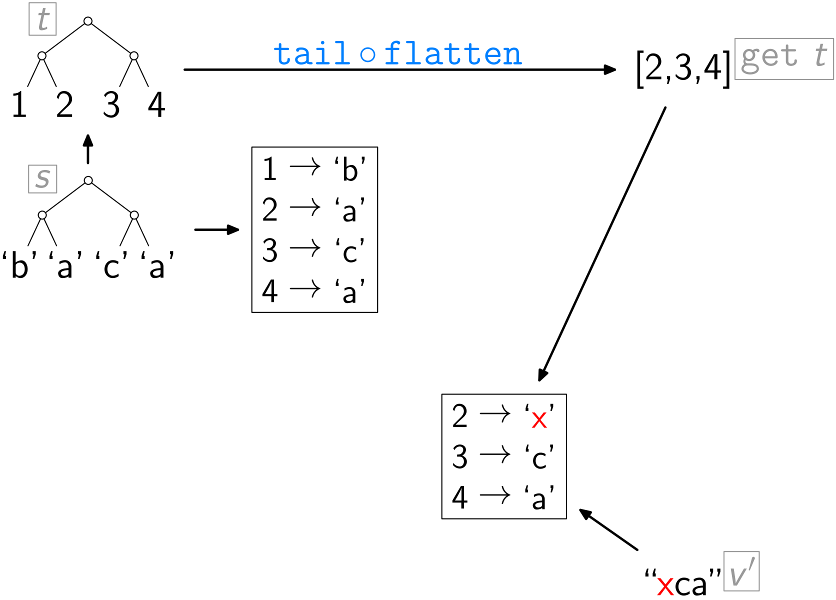

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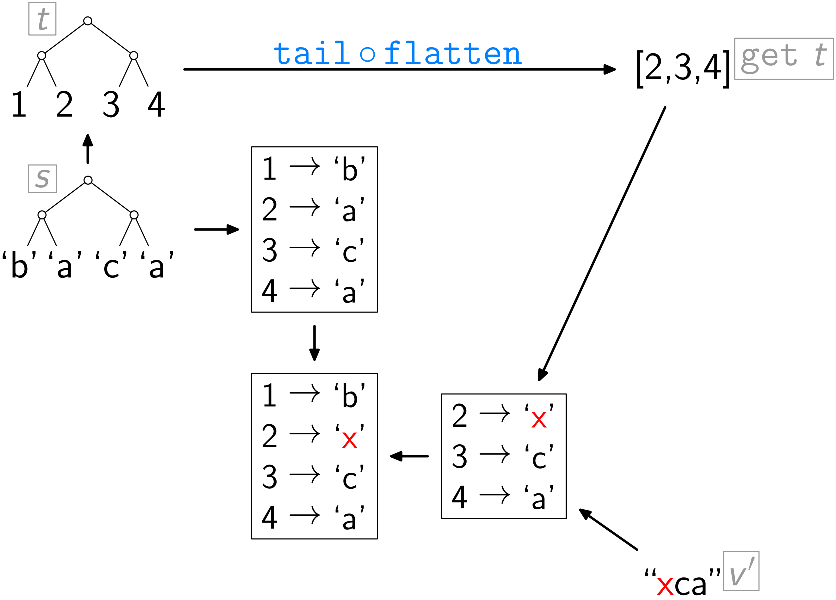

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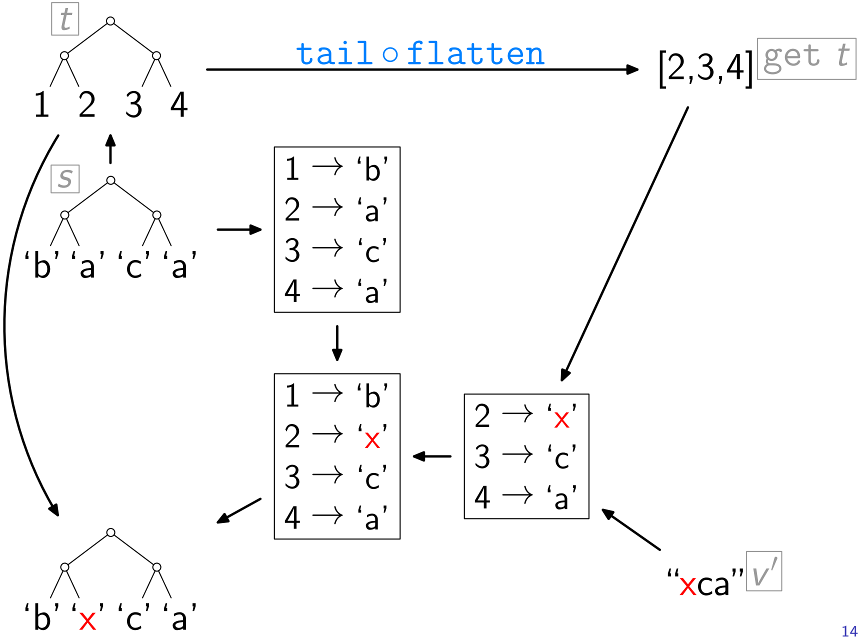

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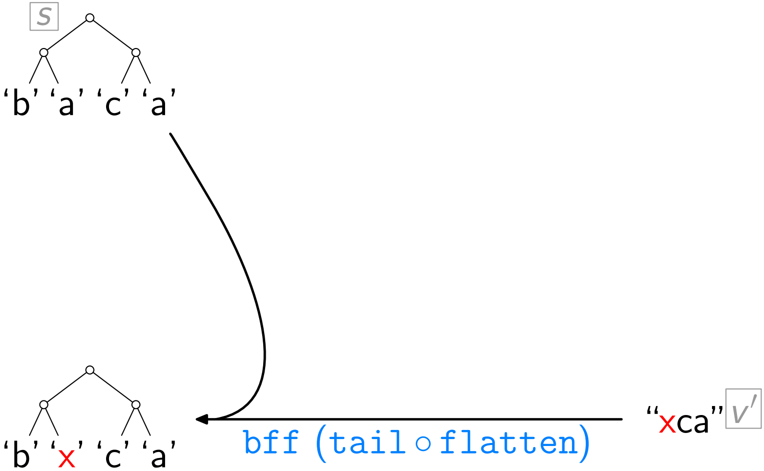

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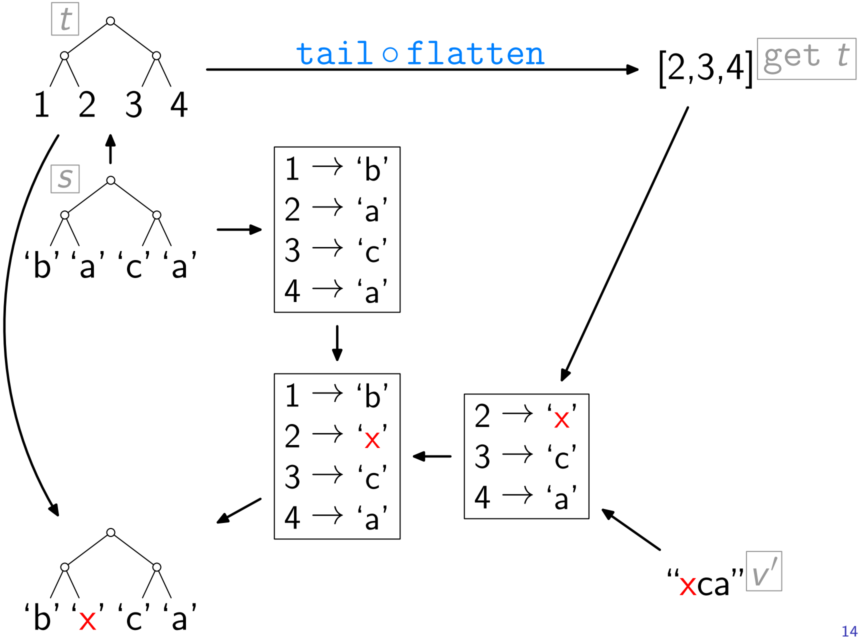

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[V. et al., ICFP'10]:

- synthesis of the two techniques
- inherits limitations in program coverage from both
- strictly better in terms of updatability than either

# References I


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