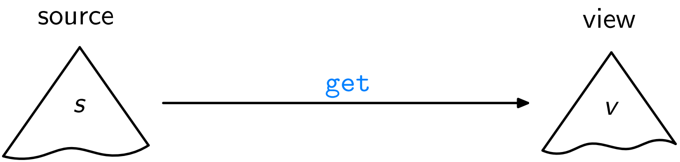
Semantic Bidirectionalisation

Janis Voigtländer

Technische Universität Dresden

April 21st, 2009

Bidirectional Transformation



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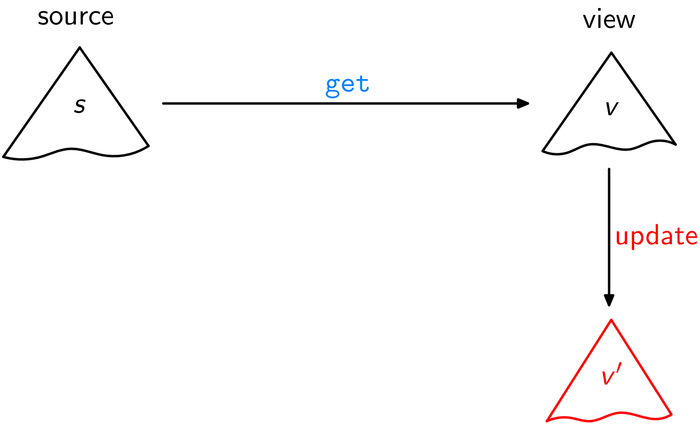

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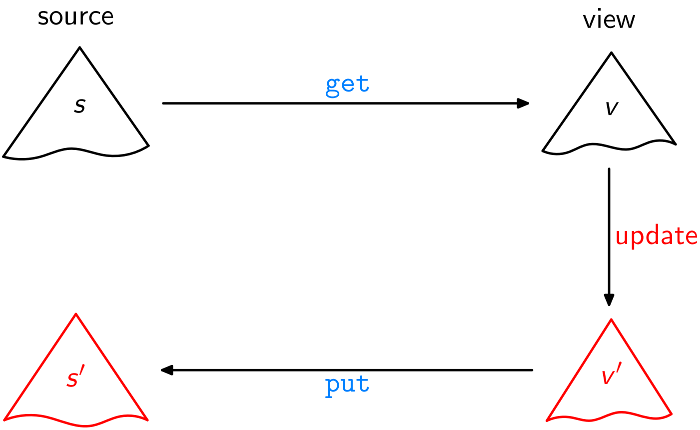

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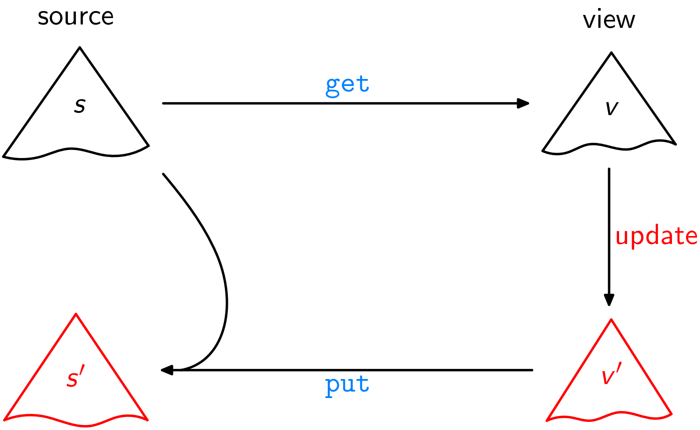

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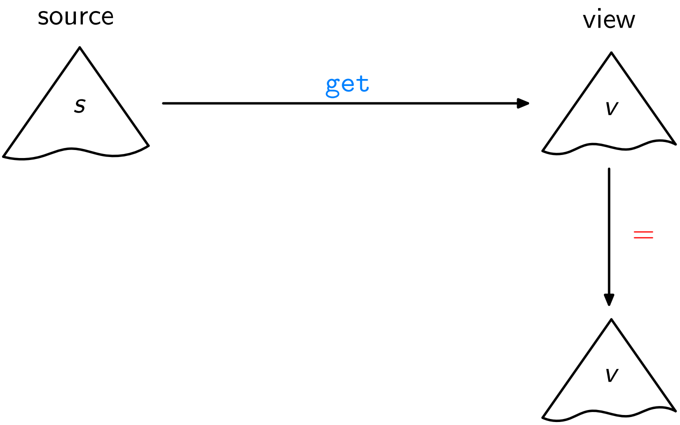

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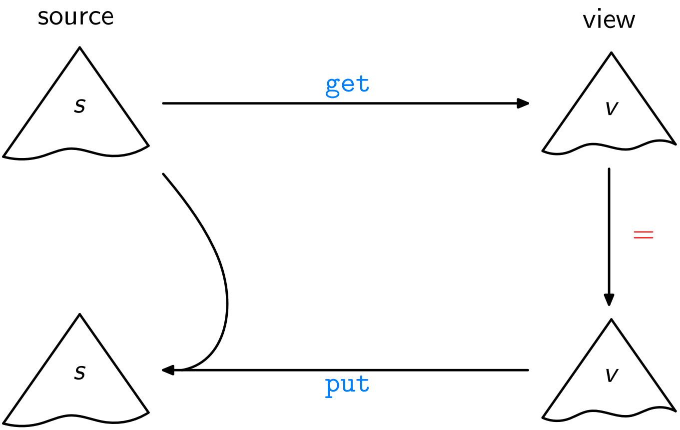

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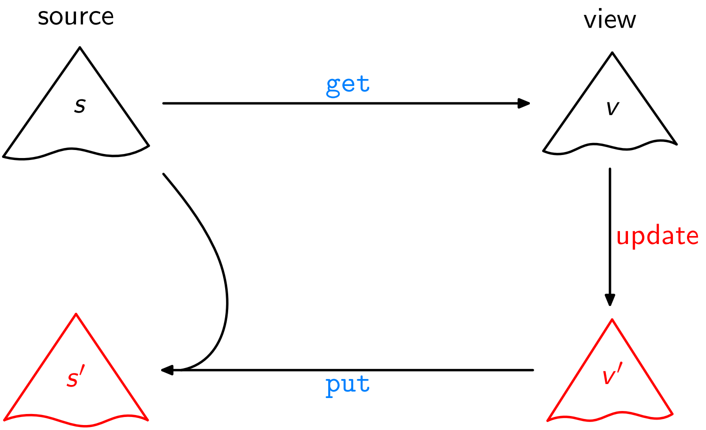

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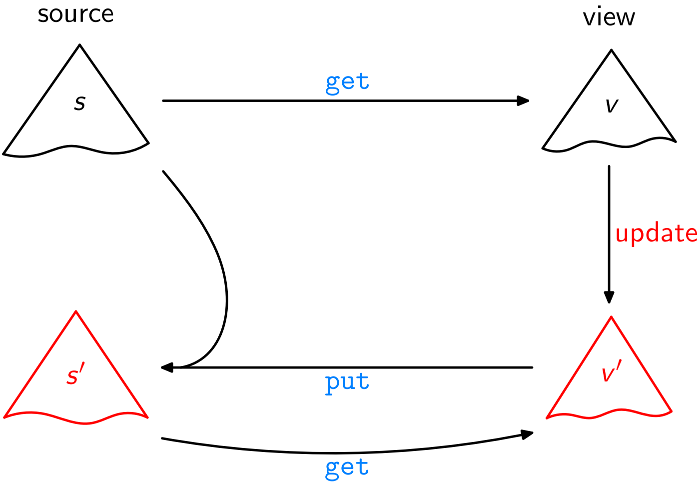

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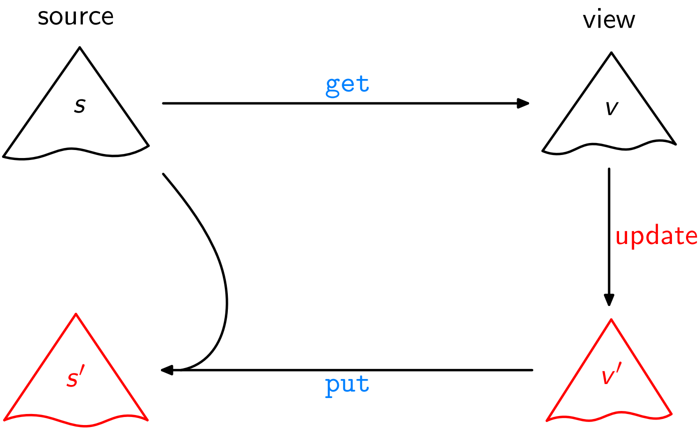

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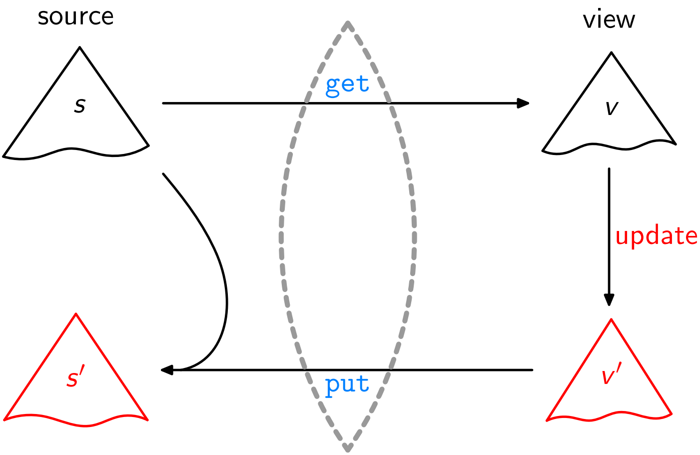

Lenses, DSLs

[Foster et al., ACM TOPLAS'07, . . .]

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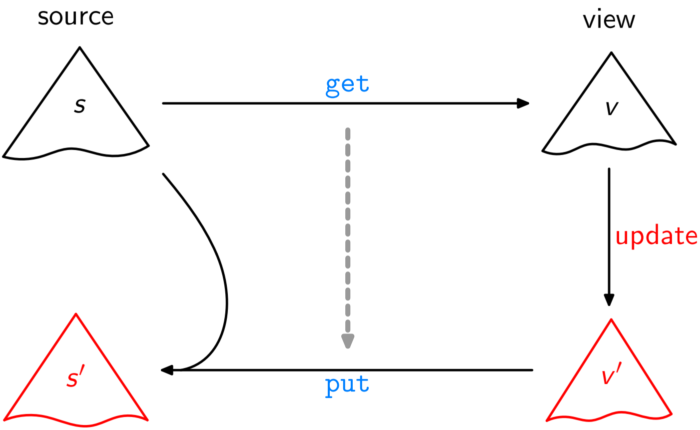

Bidirectionalisation

[Matsuda et al., ICFP'07]

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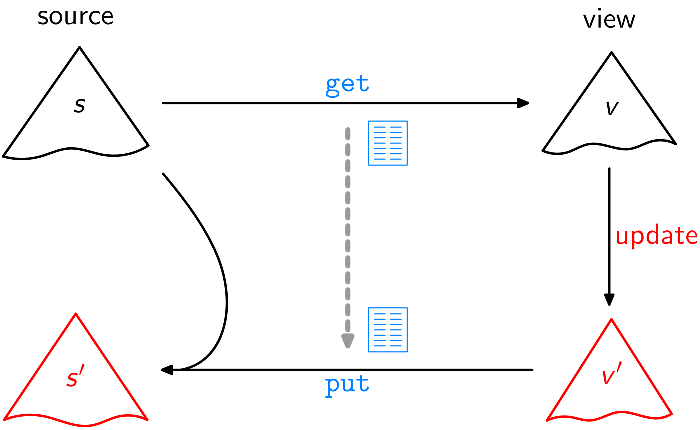

Syntactic Bidirectionalisation

[Matsuda et al., ICFP'07]

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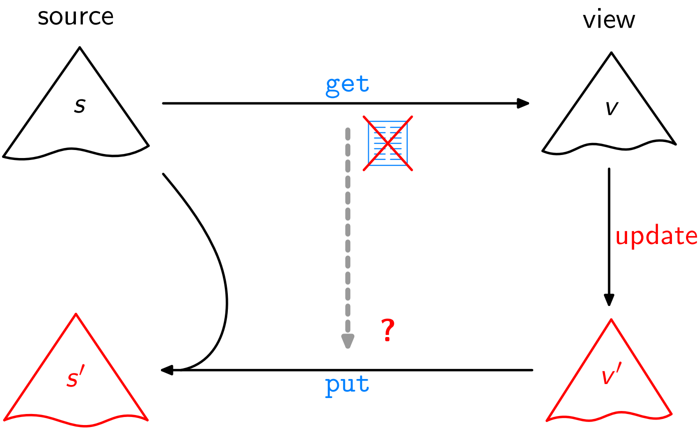

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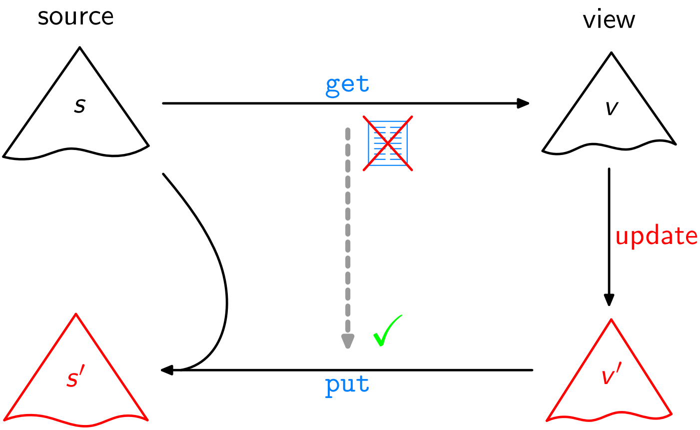

Semantic Bidirectionalisation

[V., POPL'09]

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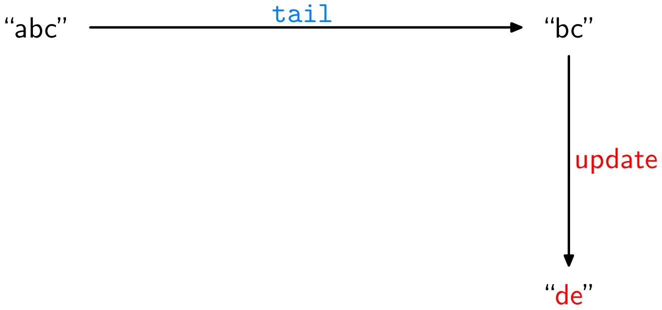

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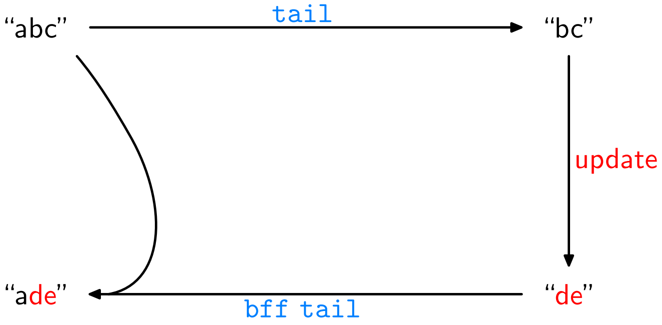

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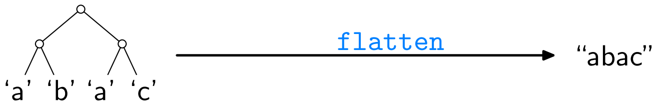

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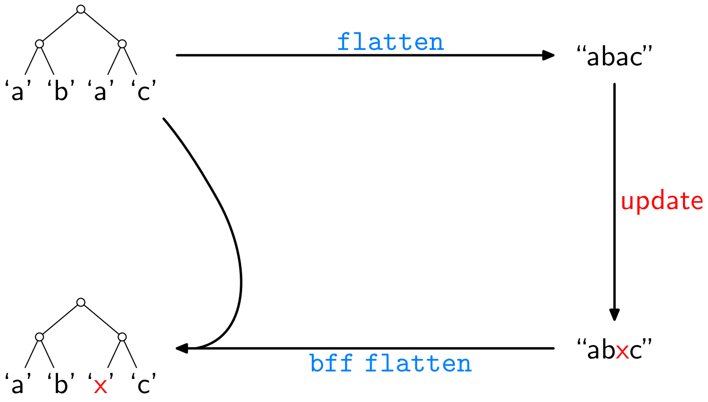

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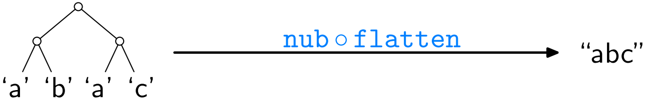

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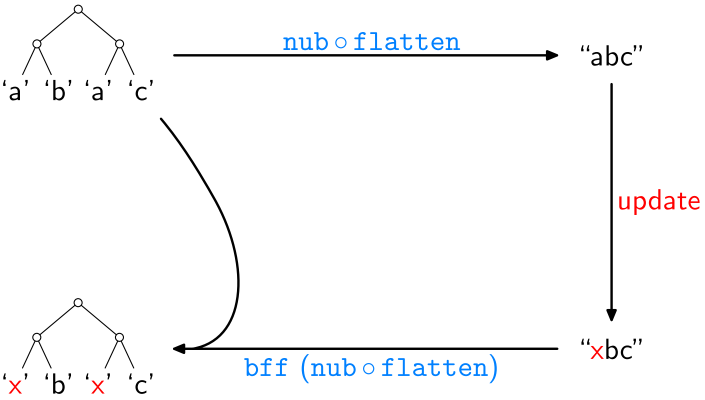

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Analysing Specific Instances

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Like:

$$\texttt{get}\ [0..n] = \begin{cases} [1..n] & \text{if } \texttt{get} = \texttt{tail} \\ [n..0] & \text{if } \texttt{get} = \texttt{reverse} \\ [0..(\texttt{min}\ 4\ n)] & \text{if } \texttt{get} = \texttt{take}\ 5 \\ \vdots & \end{cases}$$

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Then transfer the gained insights to source lists other than $[0..n]$!

Using a Free Theorem [Wadler, FPCA'89]

For every

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we have

$$\texttt{map}\ f\ (\texttt{get}\ l) \quad = \quad \texttt{get}\ (\texttt{map}\ f\ l)$$

for arbitrary f and l, where

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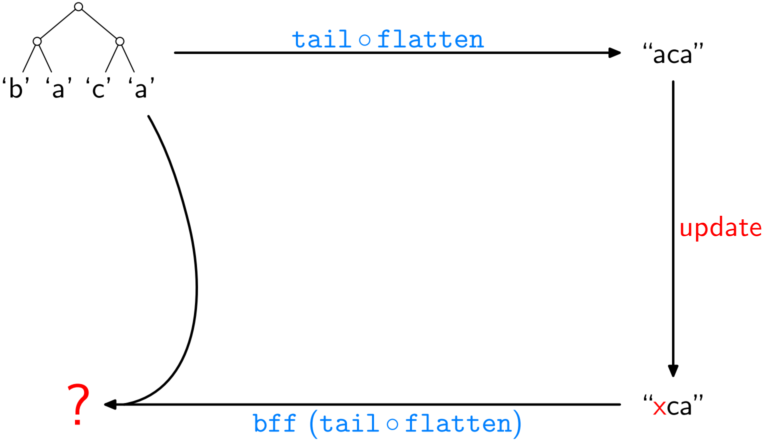

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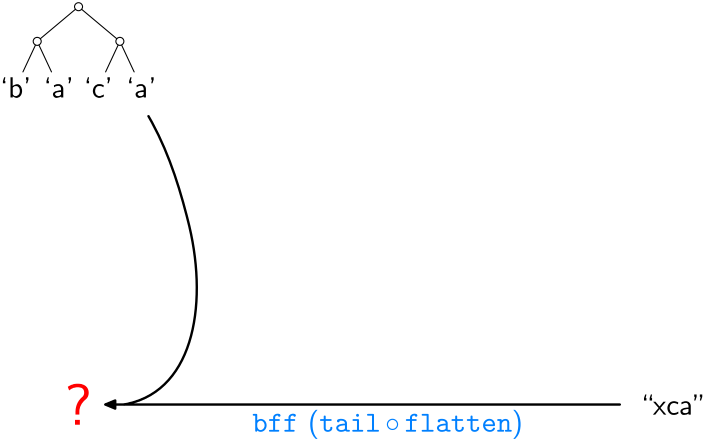

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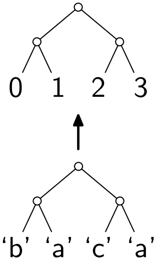

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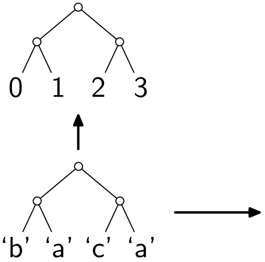

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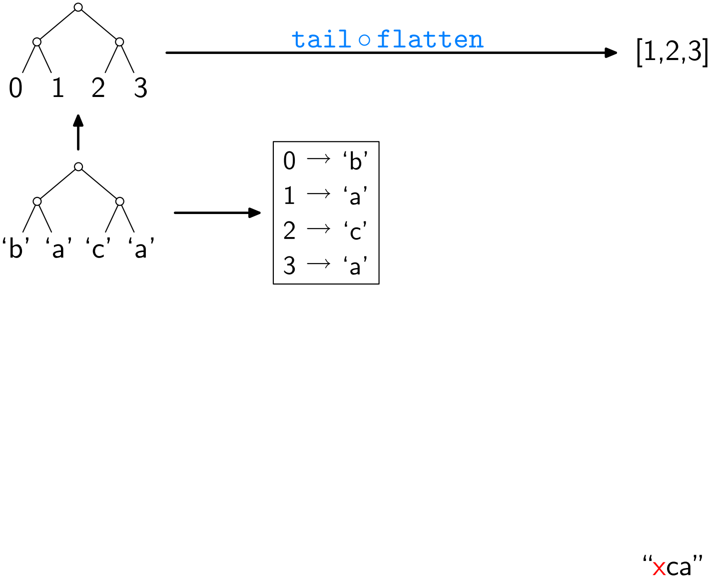

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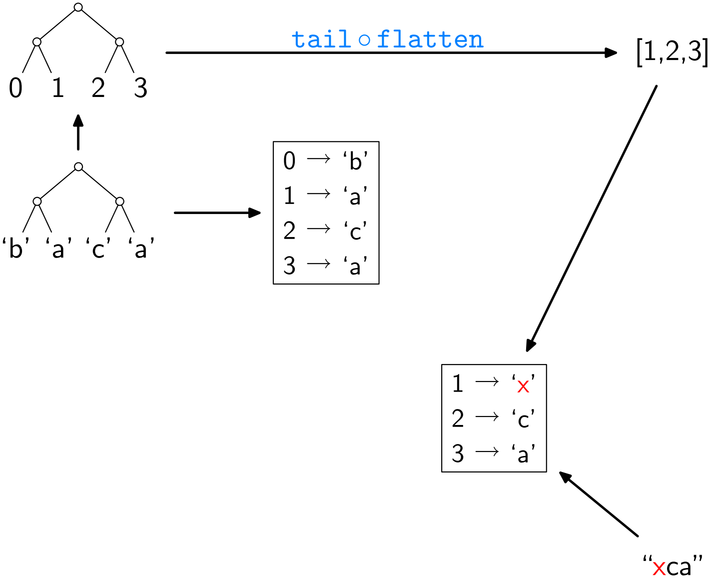

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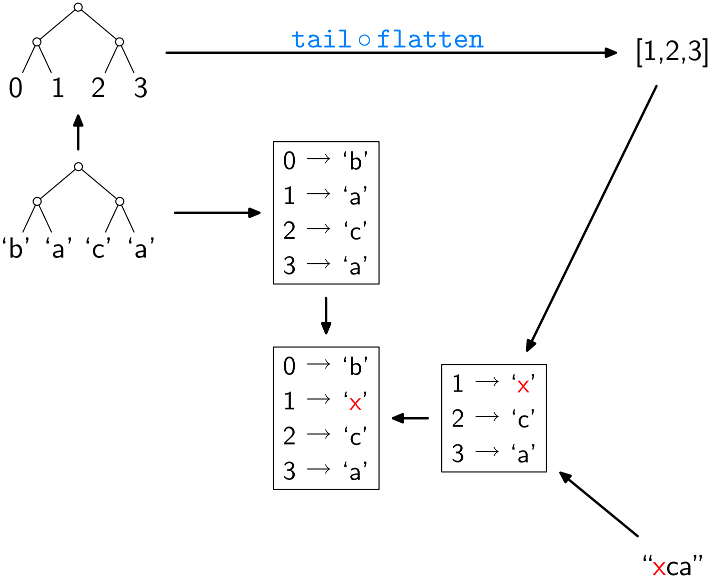

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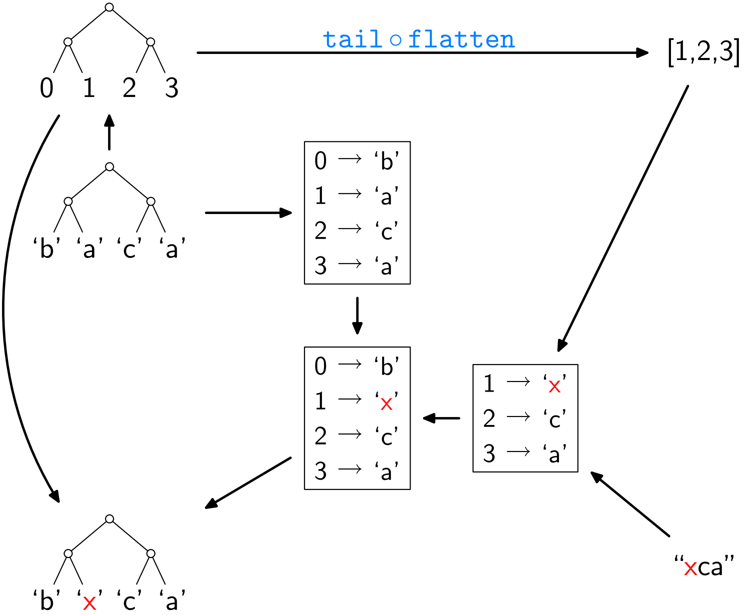

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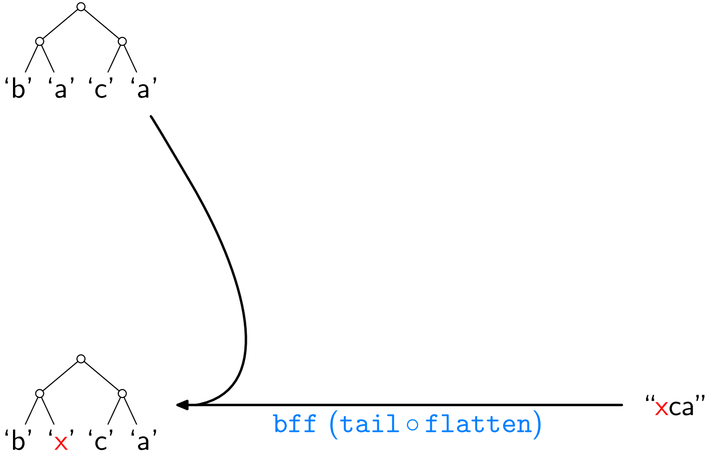

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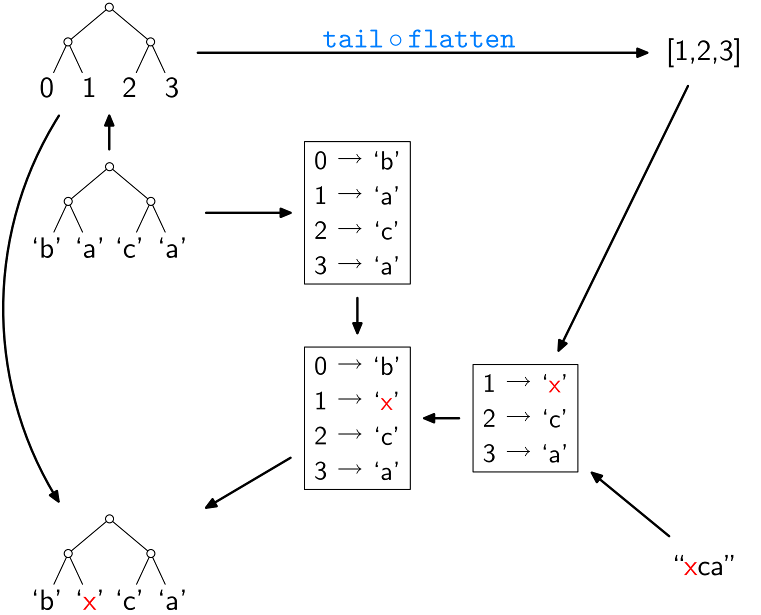

The Implementation (here: lists only, inefficient version)

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bff get s v' = let n  = (length s) − 1
                   t  = [0..n]
                   g  = zip t s
                   h  = assoc (get t) v'
                   h' = h ++ g
               in  seq h (map (λi → fromJust (lookup i h')) t)

assoc []       []       = []
assoc (i : is) (b : bs) = let m = assoc is bs
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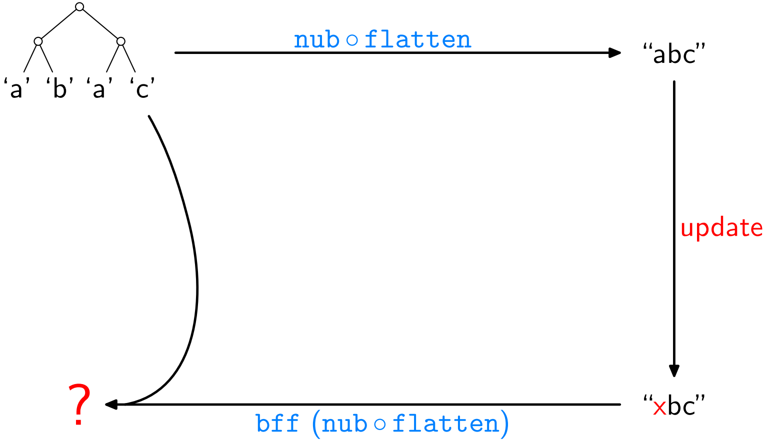
- actual code only slightly more elaborate
- online: `http://linux.tcs.inf.tu-dresden.de/~bff`

Another Interesting Example



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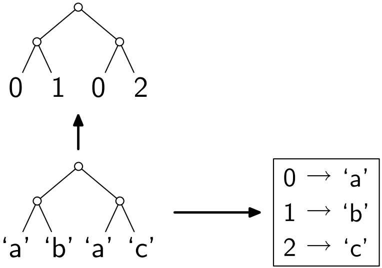

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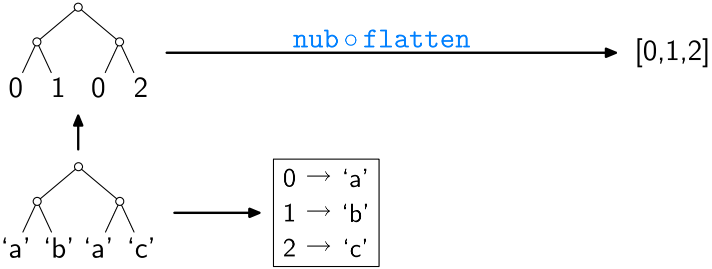

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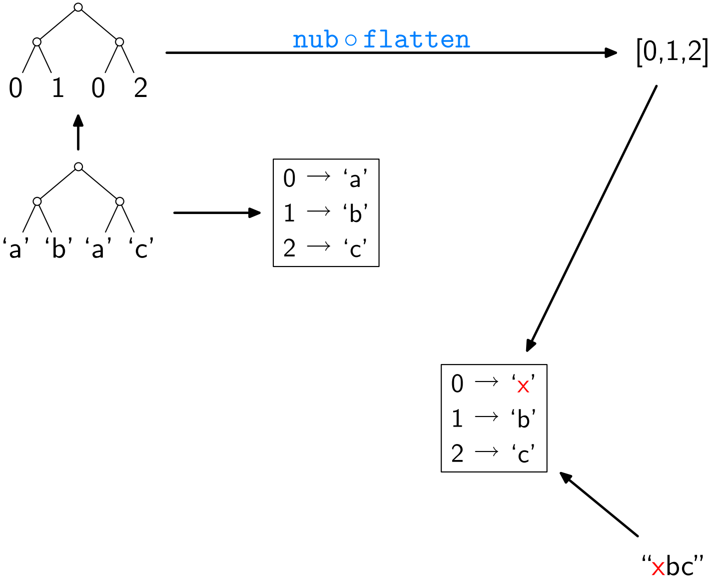

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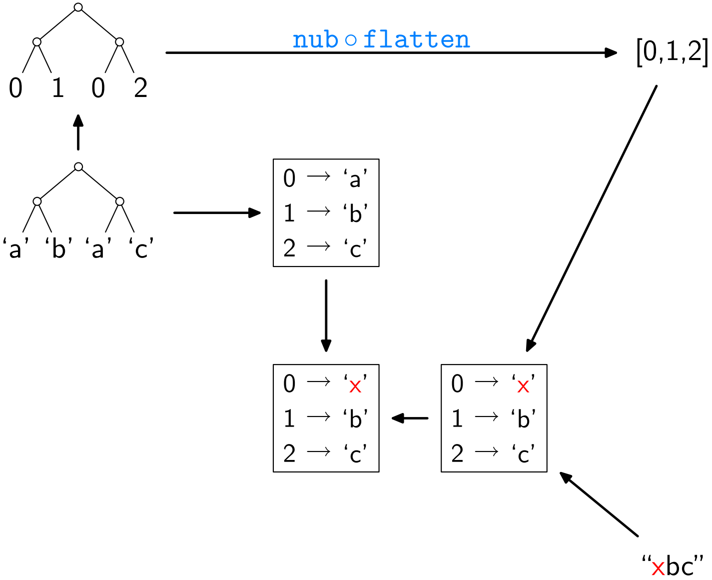

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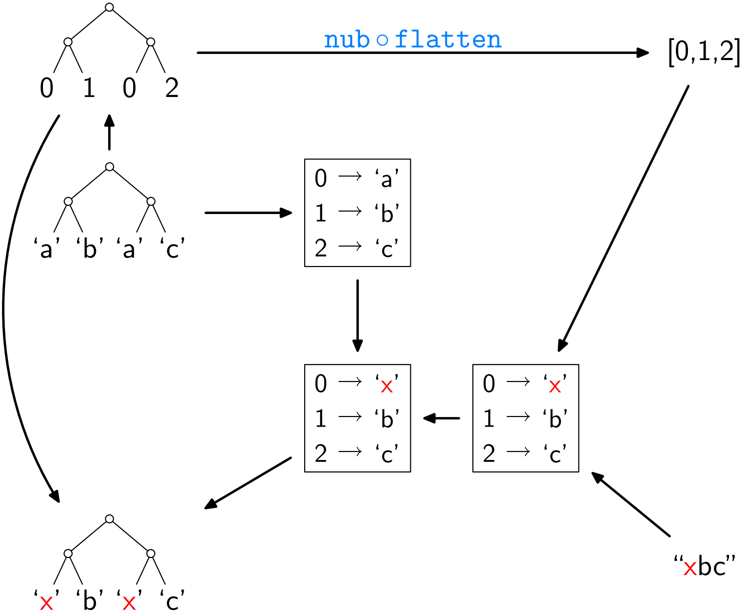

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Outlook:

- a constant-complement perspective on the method
- ... helps expanding its scope to updates that affect shape

Short Course “Free Theorems and Applications”

Three lectures, 22nd–24th April, 16.00–17.00, room IF 3.02

1. Free Theorems — Foundations
   - from intuition to a formal account
   - actually deriving free theorems
2. Knuth’s 0-1-Principle and Beyond
   - reducing algorithm correctness from infinite to finite cases
   - comparison-swap sorting and parallel prefix computation
3. Free Theorems and “Real” Languages
   - free theorems and type classes
   - free theorems and general recursion
   - automatic generation of counterexamples

References I

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