Lightweight Program Inversion

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University of Bonn

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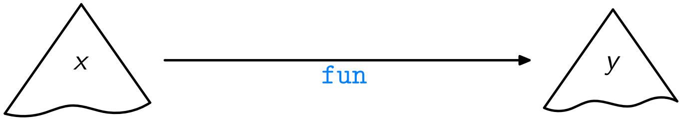
The Problem Statement



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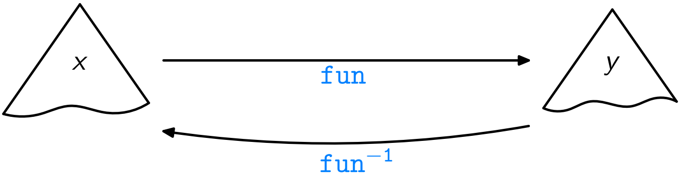

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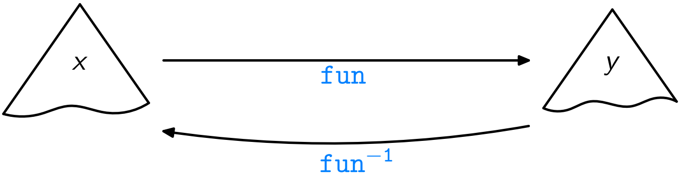

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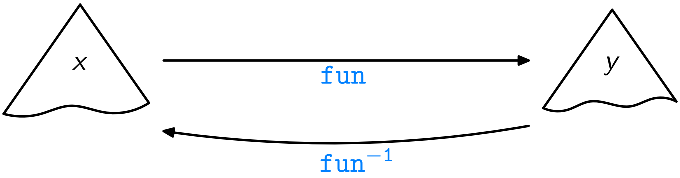

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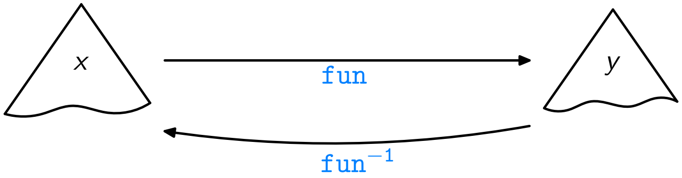

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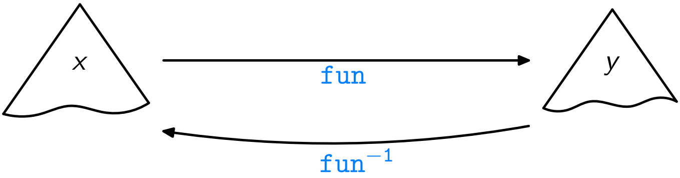

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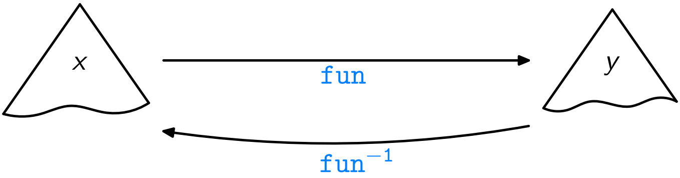

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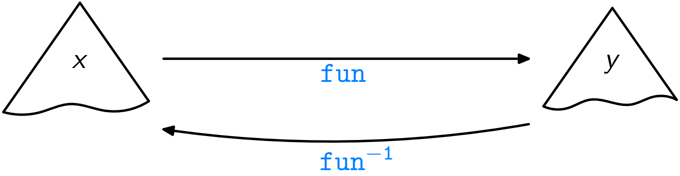

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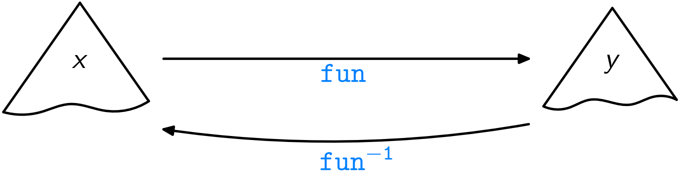

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Of course, *some* access to `fun` must be possible:

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Observation: If Bob has a winning strategy, he must be able to do without asking Alice for specific inputs up front, instead only provide a single definition of $\texttt{fun}^{-1}$ that works for all `fun`.

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- Then, an appropriate *xs* could be identified via:

$$\begin{aligned}
&\texttt{fun}^{-1} :: [a] \to [a] \\
&\texttt{fun}^{-1}\ ys = \textbf{let}\ n = \texttt{lengthInv}\ ys \\
&\qquad\qquad\qquad\ \ t\ = [1\,..\,n] \\
&\qquad\qquad\qquad\ \ h = \texttt{zip}\ (\texttt{fun}\ t)\ ys \\
&\qquad\qquad\quad \textbf{in}\ \ \texttt{map}\ (\texttt{fromJust} \circ \texttt{flip}\ \texttt{lookup}\ h)\ t
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How to implement `lengthInv`:

- If Bob had a function

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Intuition: In `checkLength`, should check that not only has $\texttt{fun}\ [1\mathinner{\ldotp\ldotp} n]$ the same length as ys, but also if two elements at positions i and j in $\texttt{fun}\ [1\mathinner{\ldotp\ldotp} n]$ are equal, then for the corresponding positions in ys, $y_i = y_j$.

Some (Necessary) Restrictions

- Need to assume that elements in *ys* can be compared. Then:

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- Need to assume that instances of Eq used are reflexive, transitive, and symmetric.

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3. Give an example of `fun` and *xs* where $\texttt{fun}^{-1}\ (\texttt{fun}\ xs) \neq xs$.

Exercises

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4. Actually prove that for every *xs*, $\texttt{fun}^{-1}\ (\texttt{fun}\ xs)$ is defined, and that whenever $\texttt{fun}^{-1}\ ys$ is defined, $\texttt{fun}\ (\texttt{fun}^{-1}\ ys) == ys$.

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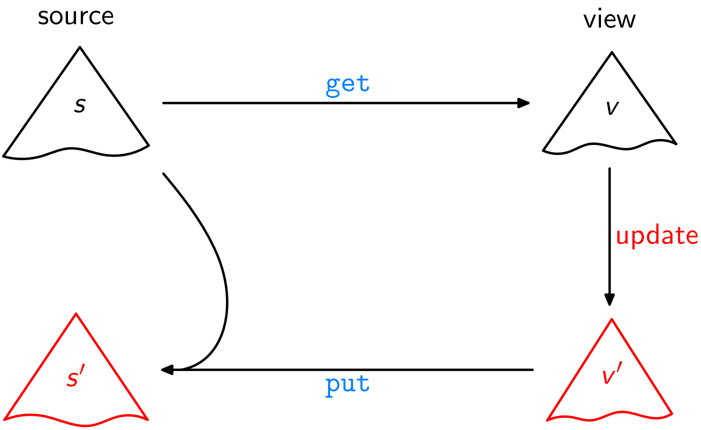
5. Generalize from lists to other data types.

Bidirectional Transformation



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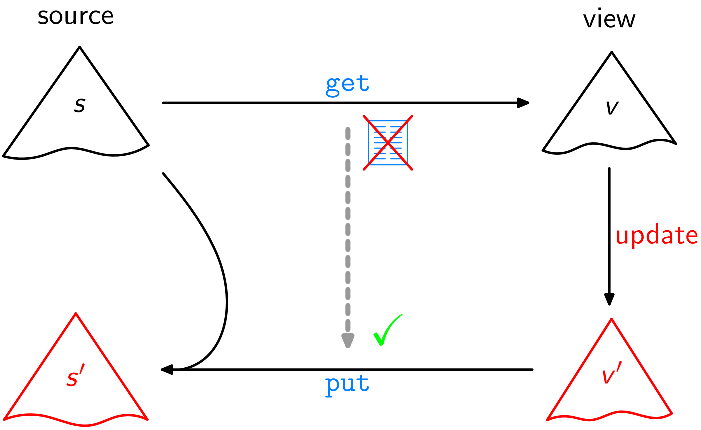

Bidirectionalization for Free!

[V., POPL'09]

HCAR

Don't forget to submit entries about your projects to the upcoming Haskell Communities and Activities Report!