Complement-Based Bidirectionalization

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Dagstuhl Seminar “bx”

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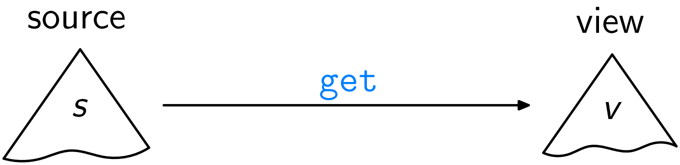
Bidirectional Transformation



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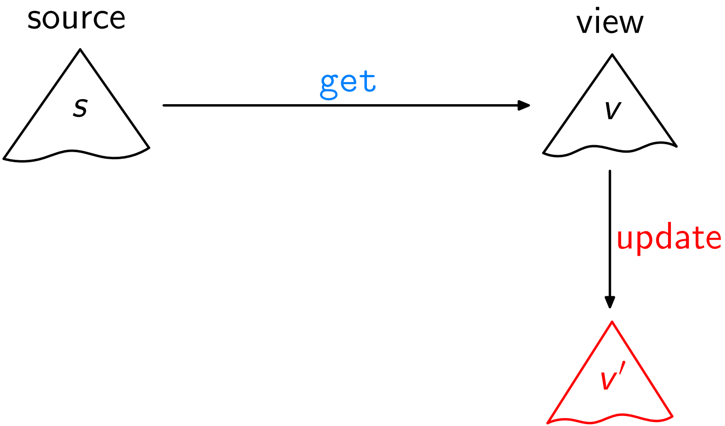

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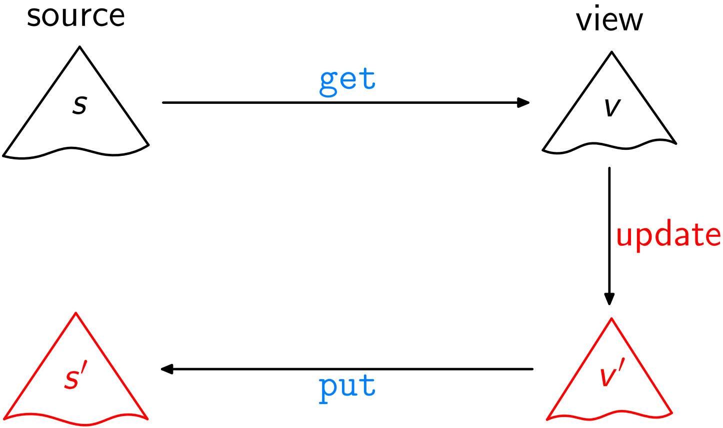

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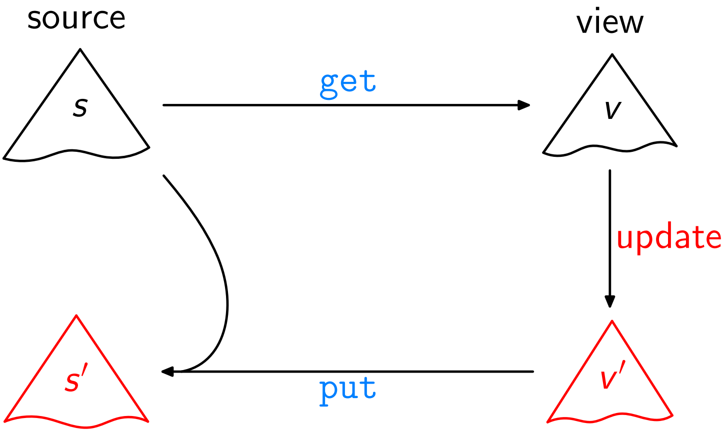

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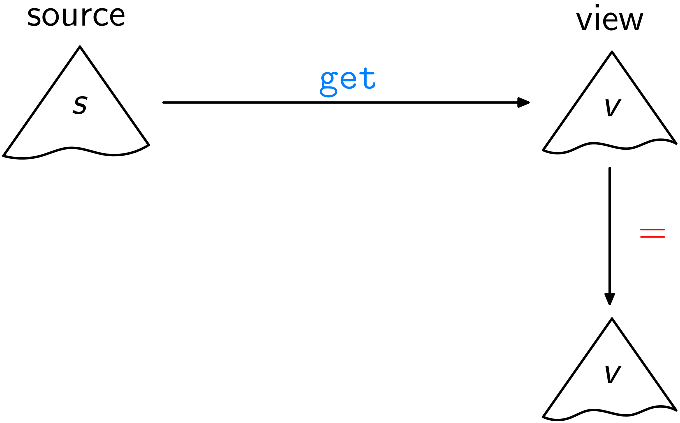

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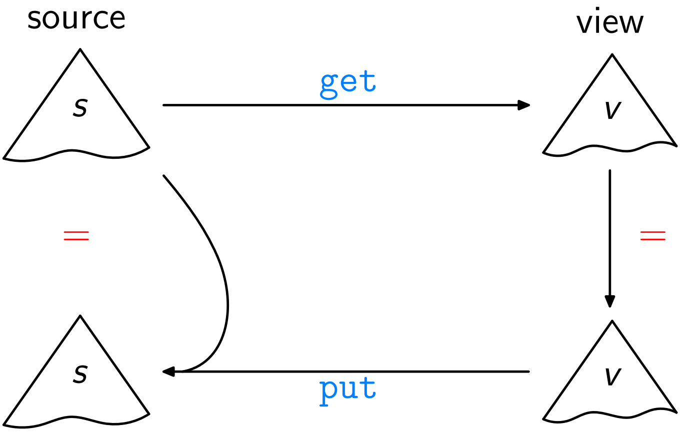

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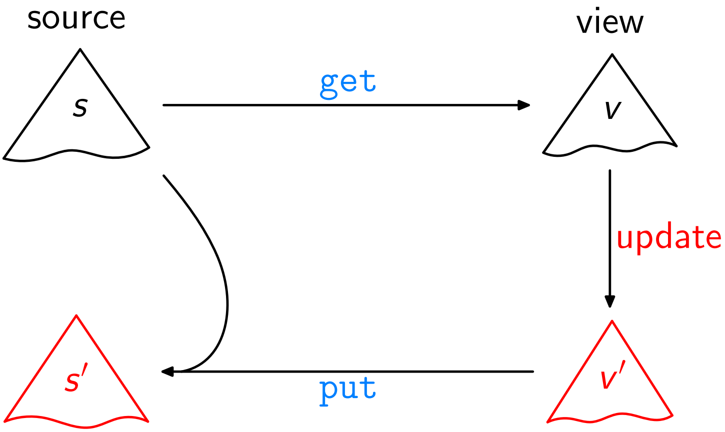

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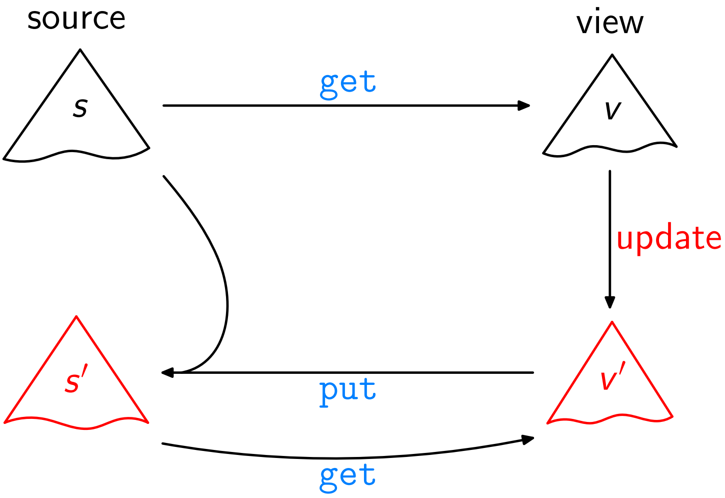

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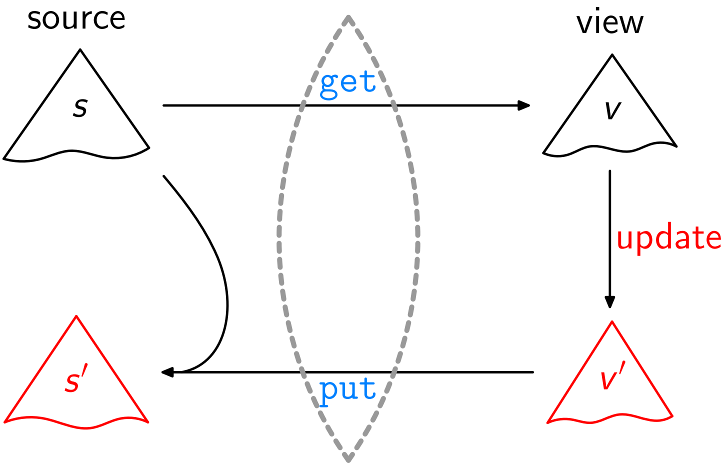

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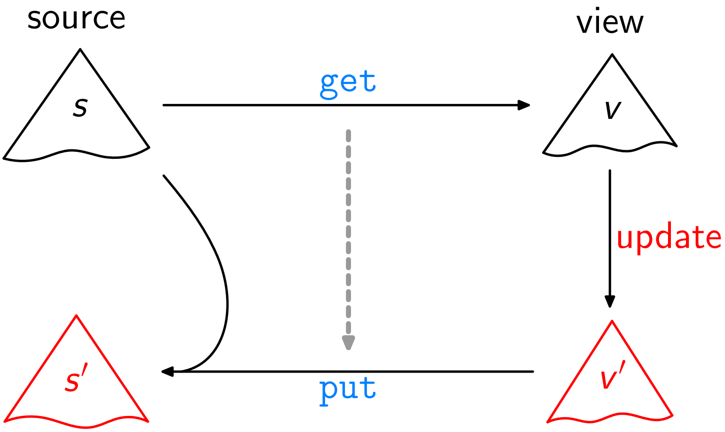

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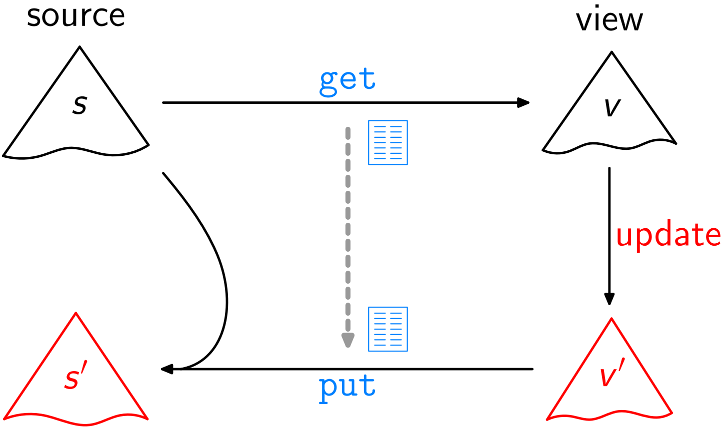

Syntactic Bidirectionalization

[Matsuda et al., ICFP'07]

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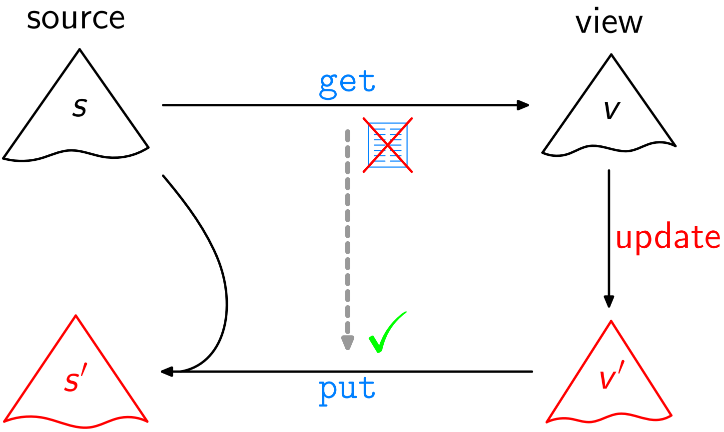

Semantic Bidirectionalization

[V., POPL'09]

The Constant-Complement Approach [Bancilhon & Spyratos, ACM TODS'81]

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Then:

$$\texttt{put} :: V \to S \to S$$
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The Constant-Complement Approach

Guarantees "reasonability":

- $\texttt{put}\ (\texttt{get}\ s)\ s = s$
- $\texttt{get}\ (\texttt{put}\ v'\ s) = v'$
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$$\begin{array}{l} \texttt{inv} :: (\mathsf{Nat}, \mathsf{Nat}_2) \to \mathsf{Nat} \\ \texttt{inv}\ (v', c) = 2 * v' + c \end{array}$$

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But allow `inv` $:: (V, C) \rightharpoonup S$, and instead of being a full inverse of `paired`, only require that:

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Guarantees (only):

- $\texttt{put}\ (\texttt{get}\ s)\ s = s$
- $\texttt{get}\ (\texttt{put}\ v'\ s) \sqsubseteq v'$
- $(\texttt{put}\ v'\ s){\Downarrow} \Rightarrow \texttt{put}\ v''\ (\texttt{put}\ v'\ s) = \texttt{put}\ v''\ s$

Choices to Make

For

$$\texttt{get} :: \mathsf{Nat} \to \mathsf{Nat}$$
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clearly

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But what about:

$$\texttt{put} :: \mathsf{Nat} \to \mathsf{Nat} \to \mathsf{Nat}$$
$$\texttt{put}\ v'\ s = 2 * v' + (v' + ((s + 1)\ \text{`}\texttt{mod}\text{`}\ 4)\ \text{`}\texttt{div}\text{`}\ 2)\ \text{`}\texttt{mod}\text{`}\ 2$$

Choices to Make

Different complement functions (`res`) lead to different update functions (`put`):

$v' \setminus s$	0	1	2	3
0	0	1	0	1
1	2	3	2	3
2	4	5	4	5
3	6	7	6	7

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In fact, `res` $:: S \rightarrow C$ is the only "handle" we have for influencing the choice of `put`.

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to be injective, but not res itself.

In fact, the "less injective", the better!

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Formally:

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Theorem [Bancilhon & Spyratos, ACM TODS'81]:
For given $\texttt{get} :: S \to V$,

$$\texttt{res}_1 \preceq \texttt{res}_2 \quad \Leftrightarrow \quad \forall v', s.\ \texttt{put}_2\ v'\ s \sqsubseteq \texttt{put}_1\ v'\ s$$

Summary of the Approach to Bidirectionalization

Given `get` $:: S \rightarrow V$, find C and `res` $:: S \rightarrow C$ such that `paired` $= \lambda s \rightarrow ($`get` $s,$ `res` $s)$ is injective and `res` is as small as possible with respect to $\preceq$.

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Define (an effective!) `inv` $:: (V, C) \rightharpoonup S$ with:

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Set:

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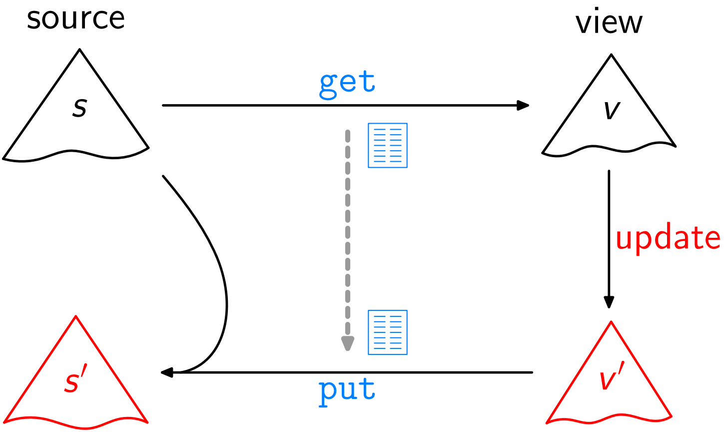

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[Matsuda et al., ICFP'07]

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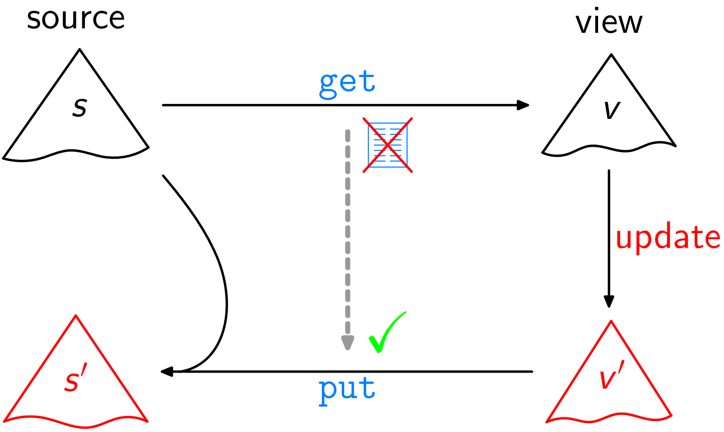

Semantic Bidirectionalization

[V., POPL'09]

Taking Stock

[Matsuda et al., ICFP'07]:

- depends on syntactic restraints
- allows (ad-hoc) some shape-changing updates

[V., POPL'09]:

- very lightweight, easy access to bidirectionality
- essential role: polymorphic function types
- major problem: rejects shape-changing updates

[V. et al., ICFP'10]:

- synthesis of the two techniques
- inherits limitations in program coverage from both
- strictly better in terms of updatability than either

Scorecard

	syntactic	semantic	combined
Update?	State-based		
Bijective?	No		
Well behaved?	Yes		
Very well behaved?	Yes		No
Choice of `put`?	No		Yes
Total?	No		

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