

# Circular vs. Higher-Order Shortcut Fusion

Janis Voigtländer

Technische Universität Dresden

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## Classical Shortcut Fusion [Gill et al., FPCA'93]

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**where** *go*  $i = \mathbf{if}\ i > n\ \mathbf{then}\ []$

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1. Write  $upTo$  in terms of  $build$ .
2. Write  $sum$  in terms of  $foldr$ .
3. Use the following fusion rule:

$$foldr\ h_1\ h_2\ (build\ g) \rightsquigarrow g\ h_1\ h_2$$

## Circular Shortcut Fusion [Fernandes et al., Haskell'07]

Producing intermediate results:

$$\begin{aligned} \mathit{buildp} &:: (\forall a. (b \rightarrow a \rightarrow a) \rightarrow a \rightarrow c \rightarrow (a, z)) \rightarrow c \rightarrow ([b], z) \\ \mathit{buildp} \ g \ c &= g \ (\cdot) \ [] \ c \end{aligned}$$



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$$\mathit{filterAndCount} :: (b \rightarrow \mathit{Bool}) \rightarrow [b] \rightarrow ([b], \mathit{Int})$$
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$$\begin{aligned} \mathit{normalise} &:: ([Int], Int) \rightarrow [Float] \\ \mathit{normalise} &= \mathit{pfold} \ \dots \end{aligned}$$

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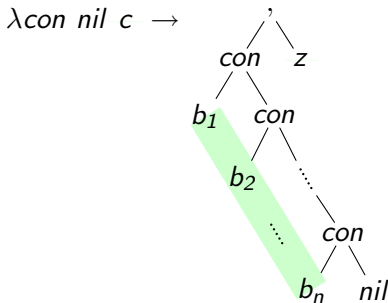
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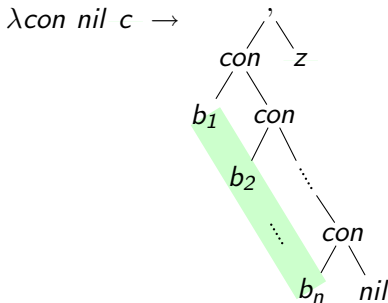


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Formal justification: free theorems [Wadler, FPCA'89]

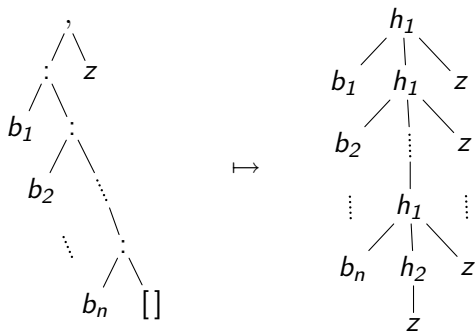


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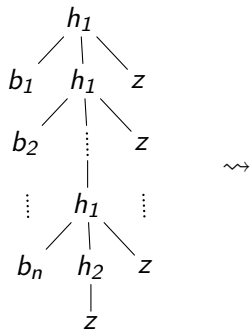
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A concrete output (*buildp g c*) will be consumed as follows:



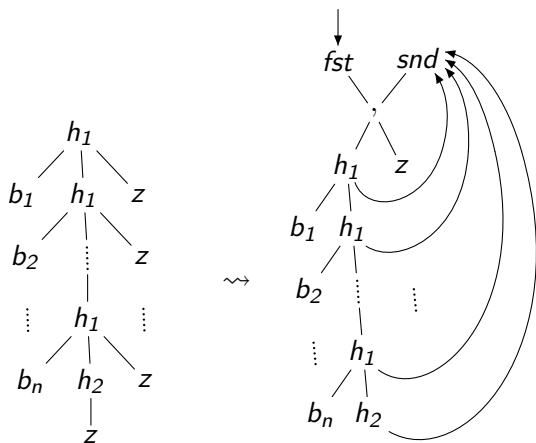
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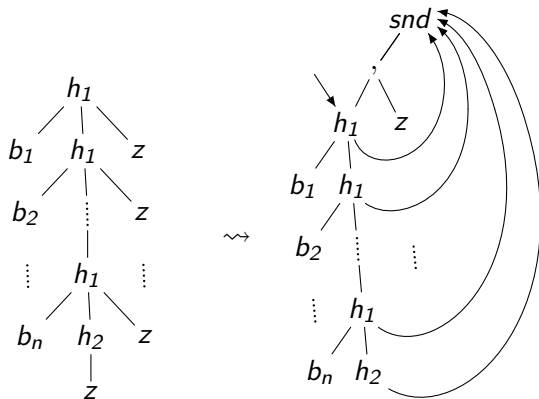
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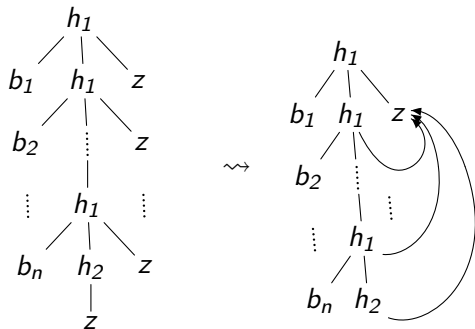
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- ▶ ...but been found to not be totally correct when considering certain language features [Johann and V., POPL'04].
- ▶ Circular shortcut fusion depends on evaluation order, which is precisely a “dangerous” corner for free theorems.
- ▶ So would it be possible to manufacture counterexamples?

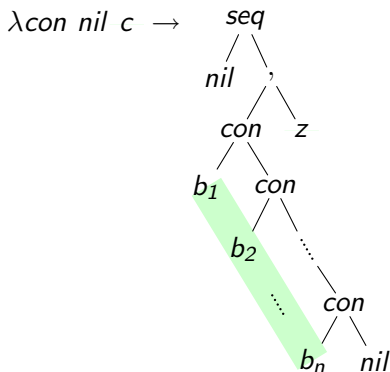
## A Problem with Selective Strictness

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In Haskell,  $g$  could also be, for example, of the following form:



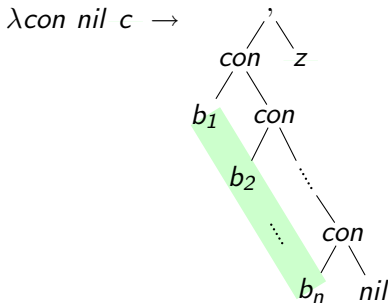
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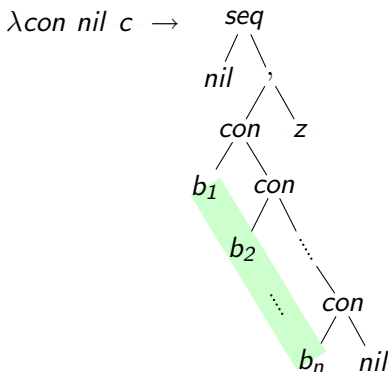
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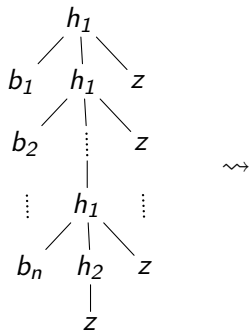
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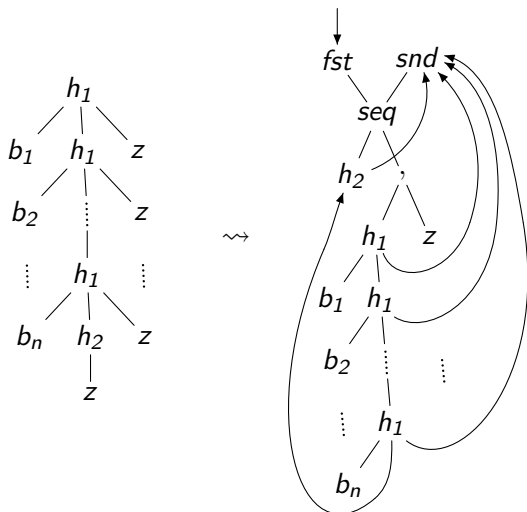
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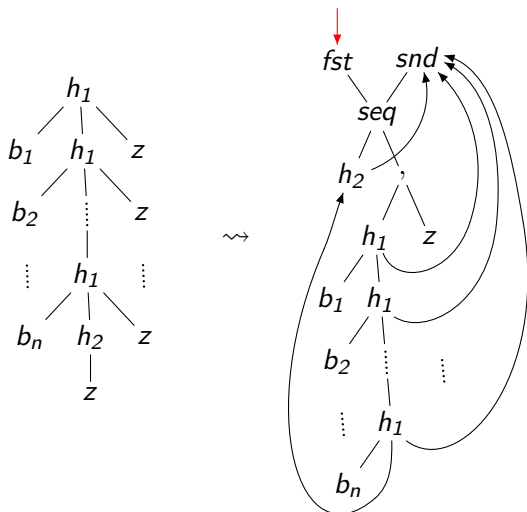
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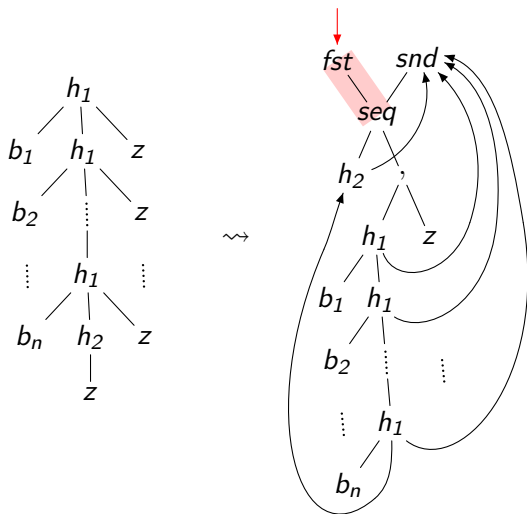
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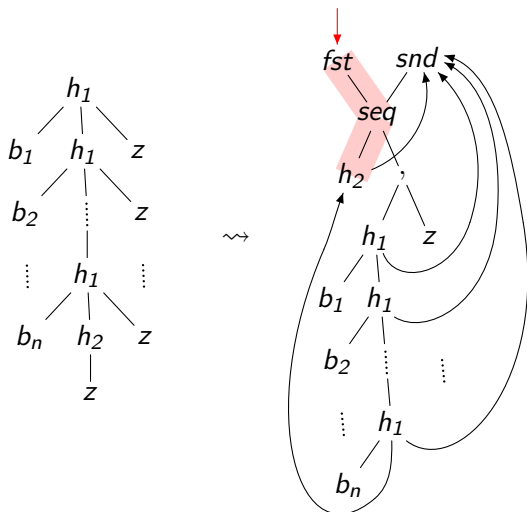
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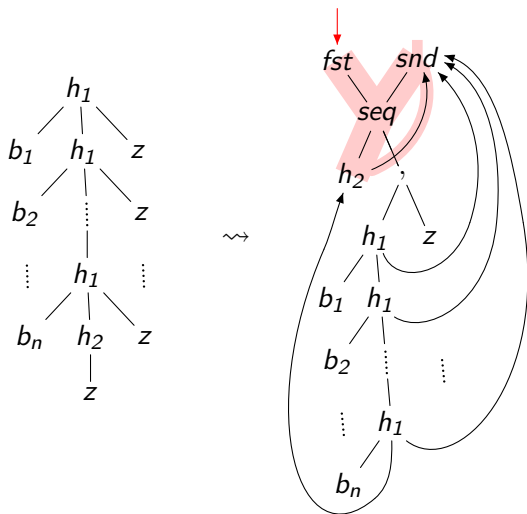
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# Total and Partial Correctness [V., FLOPS'08]

## Theorem 1

If  $h_2 \perp \neq \perp$  and  $h_1 \perp \perp \perp \neq \perp$ , then

$$\begin{aligned} & \text{pfold } h_1 \ h_2 \ (\text{buildp } g \ c) \\ & \qquad \qquad \qquad = \\ & \mathbf{let} \ (a, z) = g \ (\lambda b \ a \rightarrow h_1 \ b \ a \ z) \ (h_2 \ z) \ c \ \mathbf{in} \ a \end{aligned}$$

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## Theorem 2

Without preconditions,

$$\begin{aligned} & \text{pfold } h_1 \ h_2 \ (\text{buildp } g \ c) \\ & \quad \sqsubseteq \\ & \text{let } (a, z) = g \ (\lambda b \ a \rightarrow h_1 \ b \ a \ z) \ (h_2 \ z) \ c \ \text{in } a \end{aligned}$$

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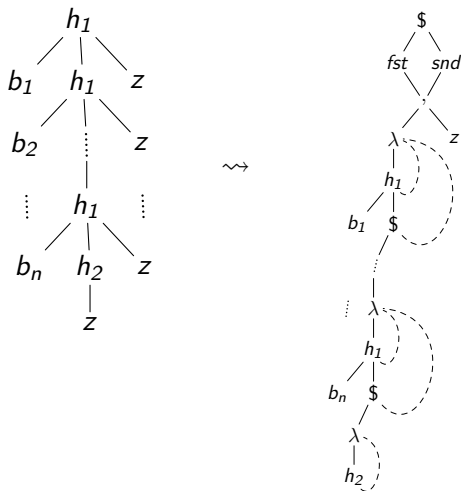
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**case**  $g\ (\lambda b\ k\ z \rightarrow h_1\ b\ (k\ z)\ z)\ (\lambda z \rightarrow h_2\ z)\ c$   
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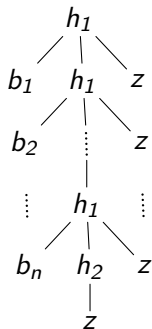
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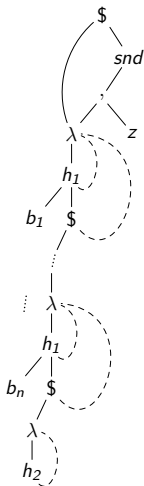


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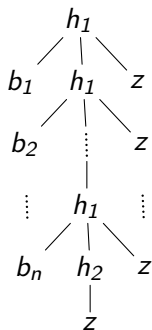


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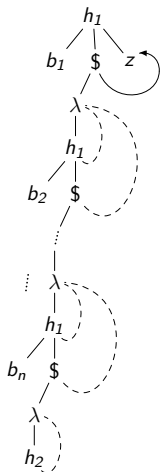


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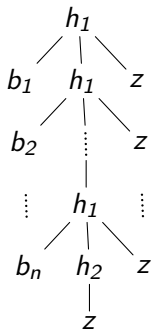


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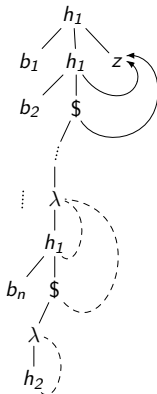


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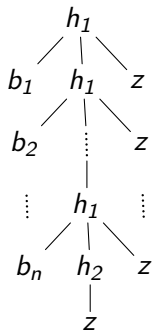


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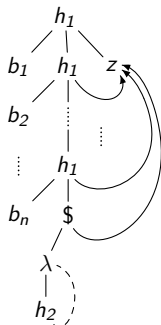


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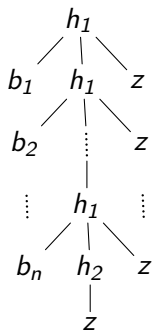


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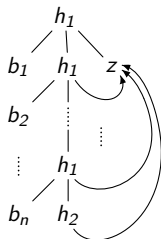


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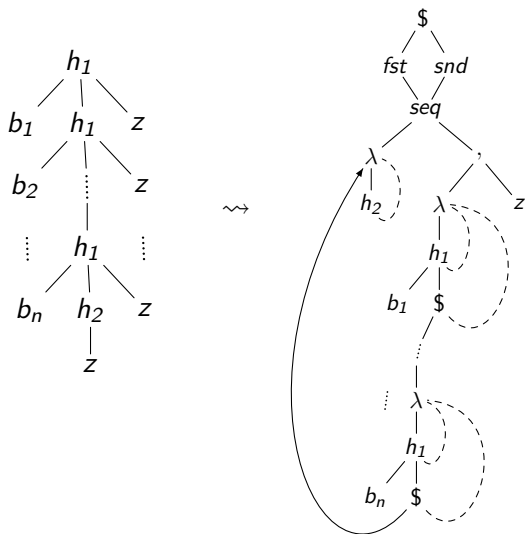


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# No Problem with Selective Strictness

For a  $g$  of the problematic form considered earlier:



# Total Correctness [V., FLOPS'08]

## Theorem 3

Without preconditions,

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- ▶ But semantically, the high-order approach is more robust.
- ▶ Performance measurements do not give a very clear picture.
- ▶ There are interesting interactions with rather low-level details of the language implementation!

## Tricky Sharing Issues — Circular Shortcut Fusion

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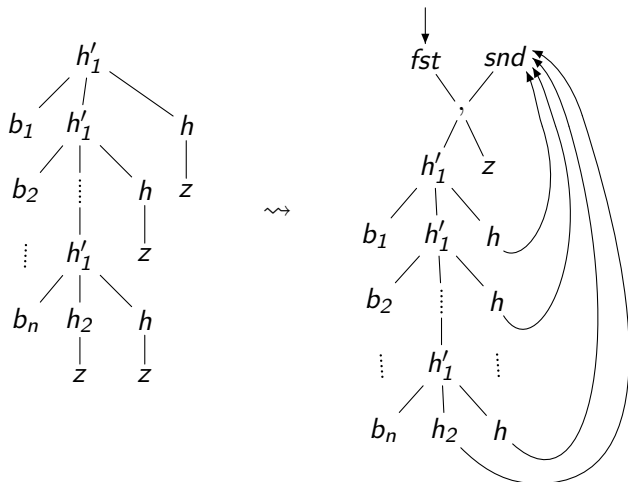
$pfold\ h_1\ h_2\ (buildp\ g\ c) \rightsquigarrow \mathbf{let}\ (a, z) = g\ (\lambda b\ a \rightarrow h_1\ b\ a\ z)\ (h_2\ z)\ c\ \mathbf{in}\ a$

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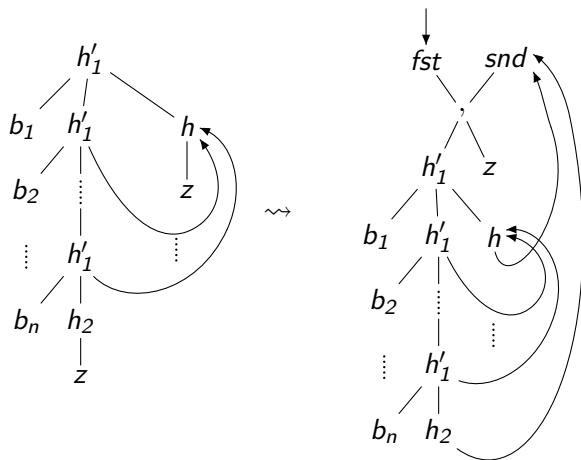




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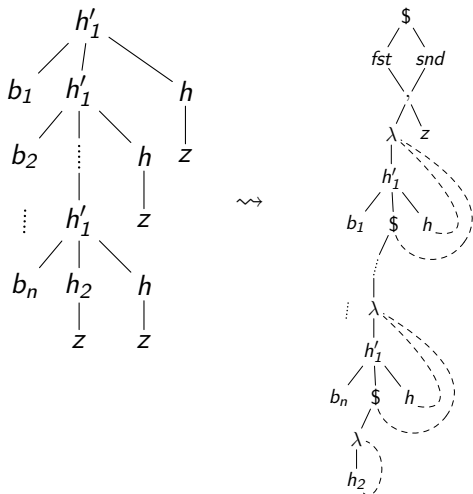
If  $h_1 = \lambda b\ a\ z \rightarrow h'_1\ b\ a\ (h\ z)$ , then **using full laziness**:



# Tricky Sharing Issues — Higher-Order Shortcut Fusion

$pfold\ h_1\ h_2\ (buildp\ g\ c) \rightsquigarrow$  **case**  $g\ (\lambda b\ k\ z \rightarrow h_1\ b\ (k\ z)\ z)\ (\lambda z \rightarrow h_2\ z)\ c$   
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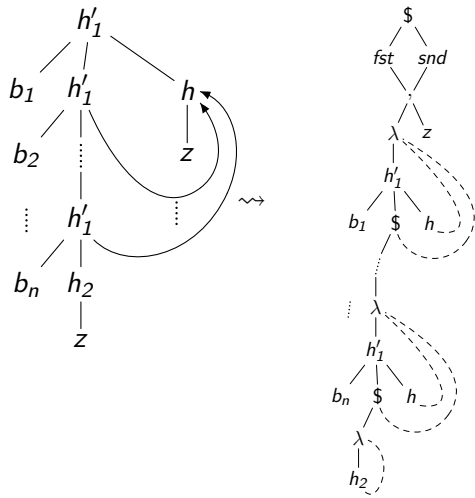
If  $h_1 = \lambda b\ a\ z \rightarrow h'_1\ b\ a\ (h\ z)$ , then:



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## What can be Learnt

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- ▶ These lessons also inform new developments for more classical shortcut fusion techniques.
- ▶ There is still an interesting design space to explore!

## Recent (and Future?) Developments

- ▶ [Pardo et al., PEPM'09] study circular and higher-order shortcut fusion in the presence of monads.

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- ▶ From a semantics perspective, the circular flavour is again more intriguing.
- ▶ The higher-order flavour is (again) more generally applicable.
- ▶ It should be interesting to investigate the interplay with other fusion work involving monads [V., MPC'08].

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